A New Full-Diversity Space-Time-Frequency Block Code for MIMO-OFDM Systems

Zahra Mohammadian\textsuperscript{1}, Mostafa Shahbinejad\textsuperscript{2}, Siamak Talebi\textsuperscript{3}
Department of Electrical Engineering, Shahid Bahonar University of Kerman, Kerman, Iran
1-Email: z.mohammadian@ieee.org
2-Email: m.shahbinejad@ieee.org
3-Email: siamak.talebi@uk.ac.ir

Received: June 2012 Revised: X XX Accepted :X XX

ABSTRACT:
Multi-input multi-output-orthogonal frequency-division multiplexing (MIMO-OFDM) is known as a proper solution for the wideband wireless communication. Numerous space-frequency block codes (SFBCs) and space-time-frequency block codes (STFBCs) have been proposed so far for implementing MIMO-OFDM systems. In this paper, a new full-diversity STFBC is proposed for two transmit antennas, which could benefit from the maximum coding advantage when delay and power profiles (DPPs) of the channel are available at the transmitter. Furthermore, simulation results confirm that the proposed STFBC outperforms the other recently proposed STFBCs with the same order of the receiver complexity.

KEYWORDS: Index Terms—Wireless communication, space-time-frequency coding, frequency-selective fading channels, multi-input multi-output-orthogonal frequency-division multiplexing (MIMO-OFDM) systems.

1. INTRODUCTION
As a sort of multi-input multi-output (MIMO) systems, space-time coding is one of the most advanced methods used to deal with ruinous fading effect of the wireless channels for the narrowband wireless communication \[1\]-\[4\]. High speed data transmission could also turn the flat-fading channels into frequency-selective channels which causes an extreme intersymbol interference (ISI) in addition to fading. Space-time coding can be used in the frequency-selective channels too \[5\]. But equalizers are needed in the receiver that follows the complexity of the receiver and the loss of the frequency diversity. Indeed, in the frequency-selective channels, due to \( L \) different paths between each pair of the transmitter and the receiver antennas, there are \( L \) different replicas of the transmitted signals at the receiver. So the communication system suffers from the ISI. This multipath phenomenon seems to be distasteful at first, but it could be used as a source of diversity \[7\], \[8\].

In order to deal with the ISI effect of the channel, orthogonal frequency-division multiplexing (OFDM) have been developed. OFDM spreads symbols over a larger time slot using orthogonal subcarriers for modulating different symbols. In fact, OFDM transforms the wideband channels to a set of narrowband flat-fading sub-channels.

Multi-input multi-output-orthogonal frequency-division multiplexing (MIMO-OFDM) systems take advantage of both OFDM and MIMO for multipath and fading channels, respectively. Space-frequency block codes (SFBCs) and space-time-frequency block codes (STFBCs) are two schemes for implementing MIMO-OFDM systems \[6\]-\[15\]. SFBCs use both the multipath and the additional spatial diversity \[5\]-\[8\]. The performance criteria of SFBCs are derived in \[7\]. For a full-diversity SFBC, the highest available diversity is equal to \( LM_tM_r \), where \( M_t \) and \( M_r \) are the number of the transmitter and the receiver antennas, respectively. The STFBCs utilize more than one timeslot by coding across multiple OFDM blocks, which presents the additional time diversity. Design criteria of STFBCs are provided in \[8\], \[10\], \[16\]. The added temporal dimension can be useful from two aspects: First, to reduce the receiver complexity of the STFBCs upon quasi-static channels \[10\], \[15\], \[17\]. Second, to use an additional diversity source when channel varies for different OFDM blocks \[8\], \[9\], \[15\]. When the channel behavior changes for different OFDM blocks, diversity gain of STFBCs is \( M_tM_rL \times \text{rank}(R_T) \), where \( R_T \) is temporal correlation matrix of the channel \[8\].

Numerous SFBCs and STFBCs have been reported to date. By using the existing space-time block codes (STBCs), primary SFBCs were designed by substituting...
time slots for the frequency subcarriers \([5]\). Based on the complex orthogonal STBCs (OSTBCs) \([2]\), full-diversity STFBCs are proposed in \([6]\) and \([10]\) which benefit from simple decoder upon quasi-static channels. It could be shown that performances of these codes degrade drastically over channels that change from one OFDM symbol to the next. Rate-\(M_0\) STFBCs and SFBCs are put forward in \([6]\) and \([13]\), respectively. In \([7]\), rate-one SFBCs are explored which benefit from the maximum coding advantage, and in \([9]\) linear transform based STFBCs and SFBCs are optimized. In \([12]\), full-diversity SFBCs are attained by repeating each OSTBCs \(\Gamma_0\) times. Thus, denoting rate of the OSTBCs by \(R_0\), they make a tradeoff between the symbol transmission rate of \(R_0\) and the diversity advantage equal to \(\Gamma_0 M_1 M_r\). In \([14]\) and \([15]\), authors presented a new class of full-diversity STFBCs and SFBCs based on the generalized block-diagonal quasi-orthogonal STBCs.

In this paper, we aim to propose a new STFBC. In the new scheme, we first generate a linear combination of symbols by using the Vandermonde matrices. Then, we substitute these combinations of symbols in the Alamouti code. It is shown throughout the paper that the newly proposed scheme attains full-diversity. The proposed code. It is shown throughout the paper that the newly proposed scheme attains full-diversity. In \([6]\) and \([10]\), authors presented a new class of full-diversity STFBCs and SFBCs based on the generalized block-diagonal quasi-orthogonal STBCs.

In this section, we describe the system model of a MIMO-OFDM system. Consider a space-time-frequency-coded MIMO-OFDM system with \(M_t\) transmit antennas, \(M_r\) receiver antennas and \(N\) subcarriers with \(K\) successive OFDM blocks. Channel impulse response during the \(k\)th OFDM block from the transmitter antenna \(i\) to the receiver antenna \(j\) is given by \([8]\):

\[
h_{ij}^k(\zeta) = \sum_{l=0}^{L-1} a_{ij}^k(l) \delta(\zeta - \zeta_l), k = 1, 2, ..., K,\]

where \(\zeta_l\)’s are delays and \(a_{ij}^k(l)\)’s are zero-mean complex Gaussian random variables with variance \(\sigma_i^2\), indicating the complex amplitude corresponding to the \(l\)th path of the \(i\)th transmitter and the \(j\)th receiver antennas in the \(k\)th OFDM block, and \(\sum_{l=0}^{L-1} \sigma_i^2 = 1\) for normalization purposes. It is supposed that there is no spatial fading correlation between antennas, and the receiver has the perfect channel state information.

Each codeword of a STFBC can be formed as a \(KN \times M_t\) matrix as below:

\[
C = \begin{bmatrix}
C_1^T \\
C_2^T \\
\vdots \\
C_K^T
\end{bmatrix},
\]

where

\[
C_k = \begin{bmatrix}
c_1^k(0) & c_2^k(0) & \cdots & c_{M_t}^k(0) \\
c_1^k(1) & c_2^k(1) & \cdots & c_{M_t}^k(1) \\
\vdots & \vdots & \ddots & \vdots \\
c_1^k(N-1) & c_2^k(N-1) & \cdots & c_{M_t}^k(N-1)
\end{bmatrix},
\]

for \(k = 1, 2, ..., K\).

In \(3\), \(c_j^k(n)\)’s are symbols or linear combinations of them which are transmitted over the \(n\)th subcarrier by the transmit antenna and the \(k\)th OFDM block. After applying an \(N\)-point inverse fast Fourier transform to each column of \(C_k\) and adding cyclic prefix, the \(l\)th column of \(C_k\) is transmitted by the transmit antenna.

The received signal at the \(j\)th receiver antenna and the \(k\)th OFDM block, after crossing from matched filter, removing cyclic prefix and performing fast Fourier transform is given by:

\[
r_j^k(n) = \sum_{l=1}^{M_t} c_i^k(n) H_{ij}^k(n) + N_j^k(n), \quad n = 0, 1, \ldots, N-1,
\]

where \(N_j^k(n)\) denotes the zero-mean additive white complex Gaussian noise corresponding to the \(n\)th frequency subcarrier and \(H_{ij}^k(n) = \sum_{l=0}^{L-1} a_{ij}^k(l)\) represents the channel frequency response at the \(n\)th subcarrier between the transmitter antenna \(i\) and the receiver antenna \(j\) and \(w = e^{-j2\pi BW N^{-1}}\), where \(BW\) is the total bandwidth of the system.

3. NEWLY PROPOSED STFBC FOR MIMO-OFDM SYSTEMS

A. Structure of the proposed STFBC:

The initial structure of the proposed STFBC could be considered as below:

\[
C_k = [G_{k,1}^T, G_{k,2}^T, \ldots, G_{k,p}^T, Z]^T \in \mathbb{C}^{N \times 2k},
\]

where...
where $P = \left[ N \right]_{2L}$, $G_{kp}$’s for $p = 1, 2, ..., P$ are matrices of size $2L \times 2$, whose constructions are mutually exclusive, and $Z$ is an $(N - 2PL) \times 2$ matrix of zeros.

First, the data symbol vector $[s_1^p, s_2^p, ..., s_{2Lk}^p]^T$ from a constellation such as BPSK or QPSK, the precoded vector $[x_1^p, x_2^p, ..., x_{2Lk}^p]^T$ could be derived from equation below:

$$
[x_1^p, x_2^p, ..., x_{2Lk}^p]^T = V[s_1^p, s_2^p, ..., s_{2Lk}^p]^T \in C^{2KL \times 1} (7)
$$

where $V \in C^{2KL \times 2KL}$ stands for the Vandermonde matrix whose parameters are selected in the same way as those of equation (3.16) of [7]. Then, $G_{kp}$’s are generated as:

$$
G_{kp} = \begin{bmatrix}
X_p^{k-1;L+1} \\
X_p^{k-1;L+2} \\
\vdots \\
X_p^{Lk}
\end{bmatrix}, \quad k = 1, 2, ..., K. \tag{8}
$$

In (8), each $X_k \in C^{2 \times 2}$ is the Alamouti code constructed by two distinct $x_i^p$’s.

In the following, we enhance the coding advantage of the proposed STFBC by adding a new parameter, namely $\gamma_{SD}$, to its design in order to attain a better performance. The structure of $C_k$ when parameter $\gamma_{SD}$ is added to the code design is changed to $C_{M}^k$ by the following equation:

$$
C_{M}^k = P_{kb}, \quad k = 1, 2, ..., K, \tag{9}
$$

where $P_{kb} = \text{diag}(P_b, Z_b)$, and

$$
P_t = \text{diag} \left( P_b, P_b, ..., P_b \right) \otimes I_2 \in C^{L \times \frac{N}{\gamma_{SD}}} \otimes I_2 \in C^{L \times \frac{N}{\gamma_{SD}}}. \tag{10}
$$

In (10),

$$
P_b = [P_1^T \ P_2^T \ \cdots \ \ P_L^T]^T \in \mathbb{N}^{\frac{N}{\gamma_{SD}} \times \frac{N}{\gamma_{SD}}}, \tag{11}
$$

where

$$
P_i = [e_i^T \ e_i^T \ \cdots \ e_i^T_{(Y_{SD}/2)-1}, i] \in \mathbb{N}^{\frac{N}{\gamma_{SD}} \times \frac{N}{\gamma_{SD}}}, i = 1, 2, ..., L. \tag{12}
$$

In (12), $e_i \in C^{1 \times \frac{N}{\gamma_{SD}}}$ is a vector whose components are all zeros except for the $i$th element that is one, and

$$
P_b = [P_1^T \ P_2^T \ \cdots \ \ P_{\frac{Y_{SD}}{2}}^T]^T \in \mathbb{C}^{1 \times \frac{N}{\gamma_{SD}} \times \frac{N}{\gamma_{SD}}} \tag{13}
$$

where $y_e = N - L\gamma_{SD} \left[ \frac{N}{\gamma_{SD}} \right]$ and

$$
P_i = [e_i^T \ e_i^T_{(1-1)}, \ \cdots \ \ e_i^T_{(\frac{Y_{SD}}{2})), i] \in \mathbb{C}^{1 \times \frac{Y_{SD}}{2} \times \frac{N}{\gamma_{SD}}} \tag{14}
$$

In (14), $e_i \in C^{1 \times \frac{Y_{SD}}{2}}$ is a vector whose components are all zeros except for the $i$th element that is one. And $Z_b$ is a $(\gamma_r - 2L \frac{\gamma_e}{2}) \times (\gamma_r - 2L \frac{\gamma_e}{2})$ matrix of zeros.

**Remark:** Regarding the structure of the proposed STFBC, its receiver complexity for the ML decoder is the same as the receiver complexity of those of [9]. In other words, when a constellation of size $M$ is employed, the receiver complexity is in the order of $O(M^{2KL})$. Also, by using the well-known sphere decoder this extent of the complexity could be decreased noticeably.

### B. Parameter $\gamma_{SD}$ Indication

In this subsection, we determine parameter $\gamma_{SD}$ so as to maximize the coding advantage of the proposed STFBC. As discussed in [8], maximizing the coding advantage of a full-diversity STFBC leads to maximizing the minimum determinant of $\Xi$ over all distinct codewords $C$ and $\mathcal{E}$, where

$$
\Xi \triangleq |\mathcal{D} \circ R| \in C^{N \times N}. \tag{15}
$$

In (15), $\mathcal{D} \triangleq (C - \bar{C})(C - \bar{C})^\dagger$ and $R \triangleq R_F \otimes R_T$, where $R_F$ and $R_T$ are the frequency and temporal correlation matrices, respectively [8]. For the maximization procedure of the coding advantage associated with our proposed STFBC to be simple, let us suppose that two distinct codewords $\mathcal{C}$ and $\mathcal{E}$ are the same except for one symbol. Supposing that this dissimilar symbol is $s_i$, by manipulation, one can easily verify that maximizing the coding advantage of the proposed STFBC presented in (9), results in maximizing the determinant of the following matrix:

$$
\bar{\Xi} = \left( I_{Lk} \otimes I_2 \right) \circ (R_T \otimes R_F). \tag{16}
$$

In (16), $R_T = W \text{diag}(\sigma_1^2, \sigma_1^2, ..., \sigma_{L-1}^2)^\dagger$ and $W \in C^{2L \times KL}$ is defined below:

$$
W = \begin{bmatrix}
1 & 1 & \cdots & 1 \\
\omega^{\ell_0} & \omega^{\ell_1} & \cdots & \omega^{\ell_{L-1}} \\
\omega^{(\ell_0+1)} & \omega^{(\ell_1+1)} & \cdots & \omega^{(\ell_{L-1}+1)} \\
\omega^{(\ell_0+2)} & \omega^{(\ell_1+2)} & \cdots & \omega^{(\ell_{L-1}+2)} \\
\vdots & \vdots & \ddots & \vdots \\
\omega^{(\ell_0+L-1)} & \omega^{(\ell_1+L-1)} & \cdots & \omega^{(\ell_{L-1}+L-1)}
\end{bmatrix} \tag{17}
$$

And $R_T$ is the temporal correlation matrix of size $K \times K$. The element associated with the $k$th row and the $p$th column of $R_T$ is obtained by $R_T(k,p) = \nu(k-p)$, where $\nu(k-p) = E\{\alpha_a^i(k)\alpha_a^j(k)\}$ [8].

Equations (16) and (17) show that the coding advantage of the proposed STFBC depends on $\gamma_{SD}$ as well as on the system bandwidth $(BW)$, DPPs and the number of subcarriers. If DPPs are known to the receiver, we can find $\gamma_{SD}$ so as to maximize the coding advantage of the intended STFBC. On the other hand, if DPPs are unknown, we design the proposed STFBC based on the permutation method introduced for the block circular delay diversity (BCDD) codes [9]. It should be also
noted that $\gamma_{SD}$ must be a multiple of two regarding size of $P$.
In the appendix, it is shown that the proposed STFBC achieve the maximum attainable diversity gain.

4. SIMULATION RESULTS
In the simulations, we considered a MIMO-OFDM system with two transmitter antennas, $N=128$ subcarriers, $BW = 1$ MHz and the length of cyclic prefix of 20 $\mu$s. We evaluated the performances of the new scheme by sketching average bit-error-rate (BER) versus average signal-to-noise-ratio (SNR). We compared the performance of the proposed code against those of the BCDD codes and optimum STFBCs, presented in [9] for the unknown and the known DPPs cases, respectively. These codes are the best STFBCs in the literature to the best of authors' knowledge.

In the unknown DPPs case, 2-ray equal power channel model with delay profile $\{0, 5\} \mu$s is considered. In the known DPPs case, the same channel model as [9] was investigated, i.e. a 2-ray equal power channel model with delay profile $\{0, 1\} \mu$s.

As can be seen in Fig. 1 and Fig. 2, regarding the BER values, our proposed STFBC outperforms the optimum STFBCs in both cases. Therefore, in order to obtain the diversity order of the proposed STFBC, without loss of generality and for the sake of simplicity, let us suppose that we have no permutation in the code. Thus, $T \in \mathbb{C}^{2L \times 2L}$ could be presented as follows:

$$T = \begin{bmatrix}
1 & -w^0 & 1 & \cdots & -w^{1-1} & 1 \\
-w^0 & w^0 & \cdots & w^{1-1} & 1 & \cdots \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\
-w^{(2L-1)-\delta} & w^{(2L-1)-\delta} & \cdots & w^{(2L-1)-\delta} & 1 & \cdots \\
\end{bmatrix} \in \mathbb{C}^{2L \times 2L}.$$

(22)

As Fig. 2 depicts performances of the BCDD code and the proposed STFBC when two antennas are employed at the receiver. As this figure shows, our proposed STFBC outdoes the BCDD code by almost 0.75 dB at BER=10$^{-4}$. Fig. 3 also demonstrates that our proposed optimized STFBC outperforms the optimum STFBC in [9] by nearly 3 dB at BER=10$^{-4}$.

APPENDIX
In this appendix, we show that our proposed codeachieves the maximum attainable diversity advantage for two transmitter antennas. Regarding Appendix I of [6], it could be shown that the proposed STFBC is full-diversity if $\prod_{k=1}^{K} \det (\Gamma_k) \neq 0$ for $x_i \neq x_j$. Now, for $K = 2$, we have:

$$\Gamma_1 \triangleq D_1 \otimes T \in \mathbb{C}^{2L \times 2L}, \quad \Gamma_2 \triangleq D_2 \otimes T \in \mathbb{C}^{2L \times 2L},$$

(18)

where

$$D_1 = \begin{bmatrix}
\sigma_0 \Delta_1 & \sigma_0 \Delta_2 & \cdots & \sigma_{L-1} \Delta_1 & \sigma_{L-1} \Delta_2 \\
\sigma_0 \Delta_2 & \sigma_0 \Delta_1 & \cdots & \sigma_{L-1} \Delta_2 & \sigma_{L-1} \Delta_1 \\
\sigma_0 \Delta_3 & \sigma_0 \Delta_4 & \cdots & \sigma_{L-1} \Delta_3 & \sigma_{L-1} \Delta_4 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\sigma_0 \Delta_{2L-1} & \sigma_0 \Delta_{2L} & \cdots & \sigma_{L-1} \Delta_{2L-1} & \sigma_{L-1} \Delta_{2L}
\end{bmatrix} \in \mathbb{C}^{2L \times 2L}$$

(19)

and

$$D_2 = \begin{bmatrix}
\sigma_0 \Delta^*_1 & \sigma_0 \Delta^*_2 & \cdots & \sigma_{L-1} \Delta^*_1 & \sigma_{L-1} \Delta^*_2 \\
\sigma_0 \Delta^*_2 & \sigma_0 \Delta^*_1 & \cdots & \sigma_{L-1} \Delta^*_2 & \sigma_{L-1} \Delta^*_1 \\
\sigma_0 \Delta^*_3 & \sigma_0 \Delta^*_4 & \cdots & \sigma_{L-1} \Delta^*_3 & \sigma_{L-1} \Delta^*_4 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\sigma_0 \Delta^*_{2L-1} & \sigma_0 \Delta^*_{2L} & \cdots & \sigma_{L-1} \Delta^*_{2L-1} & \sigma_{L-1} \Delta^*_{2L}
\end{bmatrix} \in \mathbb{C}^{2L \times 2L}.$$

(20)

By utilizing basic determinant properties, we have:

$$\det (D_2) = \det (\Gamma_1) = \prod_{k=1}^{L} \sigma_k^2 \times |\Delta_k|^{2L} \times \det (T).$$

(27)

Therefore,

$$\prod_{k=1}^{L} \det (\Gamma_k) = (\prod_{k=1}^{L} \sigma_k^2 \times |\Delta_k|^{2L} \times \det (T))^2.$$

(28)

Clearly, since
\[ \text{det}(V(t_1, t_2, \ldots, t_n)) = \prod_{\mu>v} (t_\mu - t_v) \]  

(29)

and \( \zeta_0 \leq \cdots \leq \zeta_{L-2} < \zeta_{L-1} \). \( \text{det}(T) \) is non-zero. \( f_1 \) and \( f_2 \) are of rank \( 2L \) and consequently the proposed code attain a diversity advantage of \( 4LM_r \).

5. CONCLUSION

In this paper, we introduced a novel full-diversity space-time-frequency block codes that can achieve the maximum coding advantage when partial channel side information is available at the transmitter. It was also shown by simulation results that the proposed space-time-frequency block code model outperforms the recently introduced space-time-frequency block codes.

REFERENCES


Fig. 1. BER performance for frequency-selective 2-ray equal power channel, BPSK constellation, $\nu_1 = 1, 1 \text{ bit/s/Hz}$. 

Fig. 2. BER performance for frequency-selective 2-ray equal power channel with delay spread of 5 sec, BPSK constellation, $\nu_1 = 2, 1 \text{ bit/s/Hz}$. 
BER performance for frequency-selective 2-ray equal power channel with delay spread of 1 sec, BPSK constellation, $\Gamma = 2, 1$ bit/s/Hz.