Stable Linear Bilateral Teleoperation System Employing an Impedance Control Via Estimated External Forces

Seyed Hamid Tabatabaei¹, Amir Hossein Zaeri²*, Mohammad Vahedi³
1- Department of Control Engineering, Saveh Branch, Islamic Azad University, Saveh, Iran.
Email: sh_tabatabaei@iau-tnb.ac.ir
2- Department of Electrical Engineering, Shahinshahr Branch, Islamic Azad University, Isfahan, Iran.
Email: a.zaeri@iaumajlesi.ac.ir (Corresponding author)
3- Department of Mechanical Engineering, Saveh Branch, Islamic Azad University, Saveh, Iran.
Email: vahedi@iau-saveh.ac.ir

Received: August 2019 Revised: November 2019 Accepted: January 2020

ABSTRACT:
We propose a new impedance control algorithm for delayed linear bilateral teleoperation systems. In the presented control strategy, with regard to a preferred impedance model for the master and slave robots, a special dynamic feature at the human and the master robot along with the slave robot and environment interface is proposed. In addition, external forces signals including operator and remote environmental forces are used in the controller to attain desired impedance model. A force estimation scheme is presented to remove measurement of external forces. Then, the desired impedance model is located into an appropriate sliding-mode control scheme to compensate the parameters uncertainties emerged by external force estimation errors. Then, the absolute stability criterion is used to investigate the stability of the closed-loop teleoperation system along with transparency. Consequently, the control strategy is implemented on 1-DOF robotic system as the master and slave robots. Simulation results verify the effectiveness of the presented impedance controller by using estimated external forces.

KEYWORDS: Teleoperation, Impedance Control Scheme, Time Delay, Force Estimation, Absolute Stability.

1. INTRODUCTION
In teleoperation applications, slave robot usually interacts with remote environment which is far from the operator in the master side. Therefore, an external force is exerted on the slave which is inevitable. These external exerted forces can dampen teleoperation system. In many cases, to reduce damping, the excessive forces occurred between the slave robot and remote environment should be prevented. It is obvious that avoiding any excessive forces makes proper tracking during free motion.

2. LITERATURE REVIEW
As mentioned in many previous researches, one of the main alternatives to decrease damping is using impedance control for teleoperation systems [1-3]. Impedance control strategy often controls the relation between applied external forces and velocity, therefore, it can be successfully utilized for teleoperation systems which are dealt with different environments [4-6]. Therefore, many previous researchers have employed impedance controller for teleoperation tasks due to this advantage. In [4], variable damping and stiffness have been used to reduce impact forces and boost tracking performance. In [5], in order to improve system stability, a controller has been presented to adaptively vary preferred impedance. In addition, in [6], the master robot workspace was initially constrained by employing an impedance model, and then to avoid operators fatigue operation, an adaptation was done on damping ratio of the master side. In addition, with regard to distance between the remote environment and the slave robot, the stiffness of the slave impedance has been modulated. Since the human and environment energetically work together with the master robot and slave robots respectively, therefore they affect the closed-loop system stability. However, operator and environment model have not been considered during stability analysis. Moreover, another significant point in teleoperation system is time delay which marks impact on the closed-loop system stability, but this issue has not been considered as well. It has been proved that a small time delay can cause unstable teleoperation systems, therefore this issue must be taken into account correctly [7].

Moreover, as presented in the above researches, the measurement of external forces is a key point to identify remote environment behavior. Then, according
to this identification, desired impedance parameters are proposed. Since external force measurement is really tough in different applications particularly medical ones, therefore several force estimations have been proposed to overcome this issue [8-12]. The proposed force estimation strategies could estimate forces properly; however, it was needed to identify system dynamic parameters correctly. Therefore, it is obvious that the presented force estimation algorithms are not robust against adverse uncertainties.

In this work, we propose a robust impedance control for a linear teleoperation system under presence of time delay between communication channels along with uncertainties in dynamic model. According to the desired impedance, the controller is proposed. To verify robustness of the linear teleoperation system against uncertainties, a sliding mode controller is presented as well. Moreover, a force estimation strategy is designed to get rid of external force measurement directly. Consequently, a stability condition for the closed-loop system including master/slave robots along with operator in the master side and remote environment in the slave robot side is presented. The absolute stability criterion is employed to study the stability of teleoperation system.

3. MATERIAL AND METHOD

In this section, initially the system dynamic model and some other items are defined. Then, an impedance control scheme for macro-micro teleoperation systems is proposed. A force estimation approach is designed to cope with direct measuring of external forces. Consequently, an absolute stability criterion is used to analyze closed-loop macro-micro teleoperation system.

3.1. Model Definition

In this section, we present some definitions employed in the control strategy. The master and slave robots are chosen one degree of freedom to make simpler the control process and the stability study. It should be noted that, the presented algorithm will be also related for system with $n$-degrees of freedom. Moreover, force and position scales factors are employed in the analysis and designing process, in order to use the obtained results in many various kinds of teleoperation and related applications. The time delay has been taken into account constant.

3.2. Dynamic Model for the Master and Slave Robots

We choose dynamic model for teleoperation system including the master and slave robots as a mass-damper system:

$$m_m \ddot{x}_m(t) + c_m \dot{x}_m(t) = \tau_m(t) + f_h(t)$$

Where, in the above equations $\dot{x}_m, \ddot{x}_m, \dot{x}_s, \ddot{x}_s$ are acceleration signals, velocities of the master and slave robots; in addition, $f_h, \tau_m$ are the human force and the force exerted on the environment by the slave robot respectively, and $\tau_m, \tau_s$ represent the controllers for the master and slave robots; moreover, $m_m, m_s$ are defined as the master and slave robots mass, respectively. $c_m, c_s$ show the master and slave viscous, respectively.

3.3. Delayed Signal and Scale Factors

We present a block diagram for the proposed teleoperation system in Fig. 1. As shown this figure, force and position of the master are sent to the slave robot and the environmental force exerted on the slave robot transmitted to the master robot through the communication channels. The delayed signals transmitted through the communication channels have been represented as follows:

$$\begin{align*}
x^T_m (t) &:= x_m (t - T_1), & \dot{x}_m^T (t) := \dot{x}_m (t - T_1) \\
x^T_s (t) &:= x_s (t - T_2), & \dot{x}_s^T (t) := \dot{x}_s (t - T_2) \\
\hat{f}^T_h (t) &:= \hat{f}_h (t - T_1), & \hat{f}^T_e (t) := \hat{f}_e (t - T_2)
\end{align*}$$

Where, $x^T_m (t), \dot{x}_m^T (t), \text{and} \hat{f}^T_h (t)$ are defined as velocity and position of the master as well as the estimated operator external force. $x^T_s (t), \dot{x}_s^T (t)$ and $\hat{f}^T_e (t)$ are defined as the position and velocity of slave robot as well as the estimated force applied on slave by the remote environment transmitted to the master robot. Consequently, $T_1$ and $T_2$ are defined as time delays between communication channels.

![Fig. 1. A block diagram of teleoperation system.](image)

The delayed signals after crossing of the communication channels are then scaled up/down by defined proper factors depend on special applications. Employing the mentioned scale factors, the position and velocity signals to the slave and the external force signal to the master side are used as follows:

$$x_s = k_p x_m^T, \quad f_h = k_f \hat{f}_e^T$$

Where, $k_f$ and $k_p$ are force and position scale
3.4. Control Design

We initially propose a force estimation strategy for the operator and environmental forces. The proposed force estimation approach is based on the disturbance observer [9].

3.5. Force Estimation Strategy for the Master and Slave Robots

Based on the equation (1), we propose a new definition for the operator force as follows:

\[ f_h(t) = m_m \ddot{x}_m(t) + c_m \dot{x}_m(t) - \tau_m(t) \]  

(3)

Thus, a force estimation approach as follows is proposed [9]:

\[ \dot{\hat{f}}_h(t) = -L_h \ddot{\hat{x}}_h + L_h (\ddot{\hat{m}}_m \ddot{x}_m(t) + \ddot{\hat{c}}_m \dot{x}_m(t) - \tau_m(t)) \]  

(4)

Where, \( \dddot{m} \) denotes the nominate value of the mass and damping coefficient. In addition, \( \ddot{\hat{x}}_h \) and \( L_h \) are defined as the estimated operator force and force estimation gain, respectively.

The significant drawback of the mentioned strategy is to measure acceleration signal directly. Due to the presence of noise, deriving acceleration from velocity by getting differentiation is not usually possible as well. Therefore, we define a proper auxiliary variable to disregard measurement of acceleration signal directly:

\[ z_h(t) = \dot{\hat{f}}_h(t) - \ddot{\hat{x}}_h(t) \]  

(5)

\[ \frac{d\ddot{\hat{f}}_h(t)}{dt} = L_h \ddot{\hat{m}}_m \ddot{x}_m(t) \]  

(6)

By taking derivation of the equation (5) and employing the equation (6), the new force estimation is reached:

\[ \dot{z}_h(t) = -L_h \dot{z}_h + L_h (\ddot{\hat{m}}_m \ddot{x}_m(t) + \ddot{\hat{c}}_m \dot{x}_m(t) - \tau_m(t)) \]  

(7)

Now, we consider the force estimation stability. Initially, the observer error is specified:

\[ e_h = \ddot{\hat{f}}_h - \dddot{m}_h \]  

(8)

According to (5), (7), (8), the dynamic error of observer is given by:

\[ \dot{e}_h = \dot{\hat{f}}_h - \dot{\hat{f}}_h = \ddot{\hat{f}}_h - \dddot{m}_h = \frac{d\ddot{\hat{f}}_h(t)}{dt} - L_h (\ddot{\hat{m}}_m \ddot{x}_m(t) + \ddot{\hat{c}}_m \dot{x}_m(t)) \]  

(9)

Where, \( \ddot{\hat{m}}_m = m_m - \ddot{\hat{m}}_m \) and \( \ddot{\hat{c}}_m = c_m - \ddot{\hat{c}}_m \). We present a Lyapunov function as follows to take into account the force estimation stability:

\[ V_h = \frac{1}{2} e_h^2 \]  

(10)

By taking derivative of the presented candidate lyapunov function and based on the equation (9), \( \dot{V}_h \) would be as:

\[ \dot{V}_h = -L_h e_h^2 + \dot{\hat{f}}_h e_h - e_h L_h (\ddot{\hat{m}}_m \ddot{x}_m(t) + \ddot{\hat{c}}_m \dot{x}_m(t)) \leq -L_h \dddot{m}_h e_h^2 + \gamma_m |e_h| \]  

(11)

The variation rate of \( f_h \) and presented \( (\ddot{\hat{m}}_m \ddot{x}_m(t) + \ddot{\hat{c}}_m \dot{x}_m(t)) \) are bounded:

\[ \exists y_m > 0, \forall t > 0, |\dot{f}_h| + |\ddot{\hat{m}}_m \ddot{x}_m(t) + \ddot{\hat{c}}_m \dot{x}_m(t)| < y_m \]  

Resulting:

\[ \dot{V}_h \leq -L_h \dddot{m}_h e_h^2 + \gamma_m |e_h| \]  

(12)

Where, \( \theta \in (0, 1) \). Thus:

\[ \dot{V}_h \leq -L_h (1 - \theta) e_h^2, \quad \forall |e_h| \geq \frac{y_m}{L_h \theta} \]  

(13)

Regarding to [12], it is resulted that:

\[ |e_h| \leq a_2^{-1} \left( \frac{y_m}{L_h \theta} \right) \]  

(14)

Note that, we use the presented force estimation strategy for the slave side as well:

\[ f_e(t) = -m_s \ddot{x}_s(t) - c_s \dot{x}_s(t) + \tau_s(t) \]  

(15)

\[ \dot{\hat{f}}_e(t) = -L_e \dddot{f}_e + L_e (\ddot{\hat{m}}_m \ddot{x}_m(t) - \ddot{\hat{c}}_s \dot{x}_s(t) + \tau_s(t)) \]  

(16)
3.6. Sliding-mode Impedance Controller for the Master

Dynamic actions between master and operator can be specified employing impedance control strategy. In the master side, impedance control strategy is specified by the model-based computed torque method. Now, it is supposed that the preferred impedance controller for the master robot is considered as follows:

$$
\ddot{\bar{m}}_m \ddot{x}_m(t) + \ddot{\bar{e}}_m \dot{x}_m(t) + \ddot{\bar{k}}_m x_m(t) = f_h(t) - k_f \ddot{\phi}(t)
$$

(17)

\(\bar{m}_m, \bar{e}_m\) and \(\bar{k}_m\) are defined as desired inertia, damping coefficient and stiffness, respectively and \(k_f\) is also force scale factor. This control scheme does a force control in the master side to return the environmental force at the slave side to the operator in the master side.

As mentioned before, there are parameters uncertainties in the dynamic model which should be considered in the proposed control strategy. Initially, it is assumed that the dynamic model is perfect, therefore, the following control scheme is proposed:

$$
\tau_{m1} = \left( \ddot{\bar{e}}_m - \ddot{\bar{m}}_m \dddot{\bar{m}}_m \dddot{x}_m \right) \dot{x}_m + \left( \ddot{\bar{m}}_m \dddot{\bar{m}}_m - 1 \right) \dot{f}_h - \ddot{\bar{m}}_m \dddot{\bar{m}}_m \left\{ k_f \dddot{\phi} + \dddot{\bar{k}}_m \dddot{x}_m \right\}
$$

(18)

\(\ddot{\bar{m}}_m\) and \(\dddot{\bar{m}}_m\) are the estimation of \(m_m\) and \(c_m\), respectively. Now, we design a sliding mode control scheme in order to this fact that the desired impedance control scheme is the same as the sliding surface. Therefore, it could be stated that we have obtained a robust control scheme. With regard to the defined desired impedance model, the sliding surface is as:

$$
s_m(t) = \frac{1}{\bar{m}_m} \int_0^t \dot{l}_m(t) \, dt
$$

(19)

Where \(\dot{l}_m\) is defined as:

$$
\dot{l}_m(t) = \ddot{\bar{m}}_m \dddot{x}_m(t) + \ddot{\bar{e}}_m \dot{x}_m(t) + \ddot{\bar{k}}_m x_m(t) - f_h(t) - k_f \ddot{\phi}(t)
$$

(20)

The system trajectories in the master side will desire to the sliding surface, if the sliding condition of \(s_m \leq -\mu_m s_m\) is satisfied, where \(\mu_m\) is positive constant. We add a discontinuous term to the control input in order to satisfy the desired sliding surface in the presence of uncertainties in the system. The control input with discontinuous term is as:

$$
\tau_{m2} = \tau_{m1} - K_{gm} \cdot \text{sat} \left( \frac{s_m(t)}{\varphi_m} \right) = \left( \ddot{\bar{e}}_m - \frac{\ddot{\bar{m}}_m}{\bar{m}_m} \dddot{x}_m \right) \dot{x}_m + \left( \frac{\ddot{\bar{m}}_m}{\bar{m}_m} - 1 \right) \dot{f}_h - \frac{\ddot{\bar{m}}_m}{\bar{m}_m} \left\{ k_f \dddot{\phi} + \dddot{\bar{k}}_m \dddot{x}_m \right\} - K_{gm} \cdot \text{sat} \left( \frac{s_m(t)}{\varphi_m} \right)
$$

(21)

Where, \(K_{gm}\) and \(\varphi_m\) are defined as the nonlinear gain as well as boundary layer thickness.

3.7. Sliding-mode based Impedance Control Strategy for the Master and Slave Robot

The proposed controller in the slave side is similar to the master side to a great extent. In the slave robot side, position tracking in the free motion as well as slave contact stability is affected by a preferred impedance model. Consequently, a desired impedance model for the slave robot would be:

$$
\ddot{\bar{m}}_s \ddot{x}_s(t) + \ddot{\bar{e}}_s \dot{x}_s(t) + \ddot{\bar{k}}_s x_s(t) = -f_e(t)
$$

(22)

Where, \(\bar{m}_s\), \(\bar{e}_s\) and \(\bar{k}_s\) are defined as the desired inertia, damping coefficient and stiffness, in addition \(\dddot{x} = x_s - k_p x_s^d\).

We design the control scheme for the slave like the master side. Initially, we assume that the plant models are perfect, therefore:

$$
\tau_{s1} = \left( \ddot{\bar{e}}_s - \frac{\ddot{\bar{m}}_s}{\bar{m}_s} \dddot{\bar{m}}_s \dddot{x}_s \right) \dot{x}_s + \frac{\ddot{\bar{m}}_s}{\bar{m}_s} \dddot{x}_s - k_p \dddot{x}_s
$$

(23)

Where, \(\dddot{x}_s\) and \(\dddot{\bar{m}}_s\) are the estimation of \(m_s\) and \(c_s\), respectively.

We design a sliding-mode control scheme which by using it, the desired impedance control is the same as the sliding surface, therefore, it can be stated that we have reached a robust impedance control scheme. By employing the preferred impedance of the slave robot, the sliding surface is as follows:
\[ s_s(t) = \frac{1}{\bar{m}_s} \int_{0}^{1} I_s(t) \, dt \]  \hspace{1cm} (24)

Where \( I_s(t) \) is:

\[ I_s(t) = \ddot{m}_s \dddot{x}(t) + \bar{c}_s \dot{x}(t) + \bar{k}_s \ddot{x}(t) - (\ddot{\xi}_e(t)) \]  \hspace{1cm} (25)

Note that \( \dot{s}(t) \) has not have any adverse term since the defined sliding surface is the integration of \( I_s(t) \). Note that while the dynamic model is completely known and the system is kept in the sliding mode, the system state remains completely on the sliding surface, and the slave robot demonstrates the desired behavior. When the wanted system is in the sliding mode condition, the sliding surface will be satisfied \( \dot{s}(t) = 0 \) \cite{11}. To solve \( \dot{s}(t) = 0 \) for the input control, the equivalent control would be as:

\[
\tau_{eq} = -\frac{\bar{m}_s}{\bar{m}_s} \left( \ddot{c}_s \dddot{x}(t) + \dddot{\bar{c}}_s \dot{x}(t) + \dddot{\bar{k}}_s \ddot{x}(t) \right) \\
+ \dddot{c}_s \dot{x}(t) + \dddot{\bar{c}}_s \dot{x}(t) + \dddot{\bar{k}}_s \ddot{x}(t) \\
+ k_p \bar{m}_s \dddot{x}_{m}(t) \\
\]  \hspace{1cm} (26)

Equation (26) is similar to the equation (23), the acceleration signal is omitted employing the dynamics of the master robot. The system trajectories will desire to the sliding surface, if the presented sliding condition of \( \dot{s}_s \dot{s}_s \leq -\mu_s |s_s| \) is occurred, where \( \mu_s \) is positive constant. A discontinuous term is added to the controller in order to satisfy the desired sliding surface under presence of uncertainties in the system. The controller with discontinuous term is as:

\[
\tau_{z2} = \tau_{eq} - K_{gs} \cdot \text{sat} \left( \frac{s_s(t)}{\varphi_s} \right) \\
= \left( c_s - \frac{\bar{m}_s}{m_s} c_m \right) \dddot{x}_s \\
- \frac{\bar{m}_s}{m_s} k_m x_s \\
+ \bar{m}_s k_p \left( \ddot{c}_s - \frac{\bar{m}_s}{m_s} \dddot{c}_m \right) \dddot{x}_m \\
+ \bar{m}_s k_p \left( \dddot{c}_s - \frac{\bar{m}_s}{m_s} \dddot{c}_m \right) \dddot{x}_m \\
+ \frac{\bar{m}_s}{m_s} k_p \dddot{x}_m \\
- \frac{\bar{m}_s}{m_s} k_p \dddot{x}_m \\
- K_{gs} \cdot \text{sat} \left( \frac{s_s(t)}{\varphi_s} \right) \\
\]  \hspace{1cm} (27)

Where, \( K_{gs} \) and \( \varphi_s \) are defined as the nonlinear gain as well as boundary layer thickness.

### 3.8. Stability Analysis

We utilize absolute stability criterion to analyze closed-loop system embracing operator in the master side, remote environment and communication channels. This concept is extensively employed to consider stability of the two-port teleoperation system which includes passive human and remote environment.

### 3.9. Stability Analysis of the Linear Teleoperation System

By substituting the proposed control input for the master robot (equation (21)) into the master robot dynamics and presenting these terms of \( s_m(t) \):

\[
m_m \ddot{s}_m(t) + \alpha_m(t) + K_{gm} \cdot \text{sat} \left( \frac{s_m(t)}{\varphi_m} \right) = 0 \\
\]  \hspace{1cm} (28)

Where:

\[
\alpha_m(t) = \Delta c_m \dot{x}_m(t) \\
+ \frac{\Delta m_m}{m_m} \left[ -\ddot{x}_m(t) + f_h(t) \right] \\
- \Delta f_h \\
- \left\{ k_1 (f_e - \Delta f_e) + k_m x_m \right\} \\
- \Delta f_h - \frac{m_m}{m_m} \Delta f_h \\
\Delta m_m = m_m - \bar{m}_m; \Delta c_m = c_m - \bar{c}_m \\
\]  \hspace{1cm} (29)

With regard to the mentioned uncertainties in equation (25), the boundary of the presented nonlinear gain, \( K_{gm} \) which satisfies the sliding condition, would be obtained as:

\[
K_{gm} \geq m_m (\mu_m + |\alpha_m(t)|) \\
\]  \hspace{1cm} (30)

Based on the nonlinear gain \( K_{gm} \) which satisfies equation (26), therefore the master state could be kept in the presented sliding surface. Thus, the master robot represents the desired impedance characteristic (\( I_m = \bar{m}_m \ddot{s} \equiv 0 \)) \cite{14}.

For the slave side, by substituting the proposed control input for the slave robot (equation (27)) into the slave robot dynamics and presenting these terms of \( s_s(t) \), as follow:

\[
m_s \ddot{s}_s(t) + \alpha_s(t) + K_{gs} \cdot \text{sat} \left( \frac{s_s(t)}{\varphi_s} \right) = 0 \\
\]  \hspace{1cm} (31)

Where,
\[
\alpha_s(t) = \Delta c_s \ddot{x}_m(t) + \frac{\Delta m_s}{m_s} [k_p \dot{x}_m^T(t) \ddot{m}_s + \dddot{c}_s \dddot{x}(t) - f_e((-\Delta e))] - \frac{1}{k_f} \Delta f_h + \frac{m_m}{k_f \ddot{m}_m} \Delta f_h
\]

\[
\Delta m_s = m_s - \dddot{m}_s; \Delta c_s = c_s - \dddot{c}_s
\]

Therefore:
\[
K_{gs} \geq m_s(\dot{\alpha}_s(t) + |\alpha_s(t)|)
\]

Now, we must represent the closed-loop teleoperation system in the two ports to use absolute stability concept. This intended two-port network includes two inputs as well as outputs. The defined inputs are defined as \((\dot{x}_m, \dot{x}_s)\) and the outputs are defined as \((f_h, f_e)\). There is a matrix which represents the relationship between inputs and outputs of the two-port. The attained desired matrix so-called hybrid matrix is shown as follows:

\[
\begin{bmatrix}
F_h \\
-V_e
\end{bmatrix} = \begin{bmatrix}
h_{11} & h_{12} \\
h_{21} & h_{22}
\end{bmatrix} \begin{bmatrix}
\ddot{m}_m \\
\ddot{m}_s
\end{bmatrix}
\]

(34)

It cannot utilize absolute stability concept directly because of this fact that the estimated forces are used in the controller. Consequently, according to the presented force estimation strategy, a relationship between \(\dddot{f}\) and \(f_e\) in the case of Laplace transform is derived as follows:

\[
\dddot{F}_h = \frac{L_h F_h}{s + L_h}
\]

\[
\dddot{F}_e = \frac{L_e F_e}{s + L_e}
\]

(35)

By substituting equation (35) into closed-loop dynamics, it can be derived, as a hybrid matrix, as follows:

\[
\begin{bmatrix}
h_{11} & h_{12} \\
h_{21} & h_{22}
\end{bmatrix} = \begin{bmatrix}
\dddot{m}_m s^2 + \dddot{c}_m s + k_m \\
-L_h k_p e^{-\tau_2 s}
\end{bmatrix} \begin{bmatrix}
\dddot{e}_s s + k_s
\end{bmatrix}
\]

(36)

The sufficient and necessary conditions to analyze stability based on absolute stability concept are as follows [13]:

- \(h_{11}\) and \(h_{22}\) should not include poles in the right half plane.
- Any poles of the \(h_{11}\) and \(h_{22}\) should be simple with real and positive residues on the imaginary axis.
- For every real values of \(\omega\):
  \[
  R_e[h_{11}] \geq 0, R_e[h_{22}] \geq 0
  \]
  \[
  f(\omega) = -\cos(\angle h_{12}h_{21}) + 2 \frac{R_e[h_{11}]R_e[h_{22}]}{|h_{12}h_{21}|} \geq 1
  \]

These mentioned conditions are called Liewellyn’s stability conditions as well. Therefore, if the presented h-parameters related to hybrid matrix can satisfy Liewellyn’s stability criterion, the closed-loop linear bilateral teleoperation will be absolute stable. Regarding to equation (36), the desired impedances and force estimation gains should be chosen to satisfy Liewellyn’s stability conditions.

4. EXPERIMENTAL RESULTS

We have done several simulations in the MATLAB/Simulink in order to evaluate the presented control scheme. We implement the control strategy on one DoF robots as the master and slave robots. Note that, the performance of the force estimation algorithm and the impedance control scheme is evaluated under presence of parameters uncertainties in the master and slave robots’ dynamics model. The block diagram of the designed control strategy has been presented in the Fig. 2.

![Fig. 2. A block diagram of the control scheme.](image)

We have determined impedance parameters and force estimation gains before simulation process.

As mentioned in the last section, we must choose these parameters to satisfy absolute stability conditions. Therefore, the impedance parameters and force estimation gains are used as illustrated in Table 1:

<table>
<thead>
<tr>
<th>Master robot</th>
<th>Slave robot</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_m = 0.2)</td>
<td>(k_s = 20)</td>
</tr>
<tr>
<td>(c_m = 0.5)</td>
<td>(c_s = 0.7)</td>
</tr>
<tr>
<td>(\dddot{m}_m = 0.1)</td>
<td>(\dddot{m}_s = 0.0125)</td>
</tr>
<tr>
<td>(L_h = 10)</td>
<td>(L_e = 10)</td>
</tr>
</tbody>
</table>

We initially generate a desired path at the master robot side for comparison of two controllers (nominal impedance controller and sliding-mode one in the presence of estimated external forces) (Fig. (3)).
As illustrated in Fig. 3, free motion occurs between 10 to 18 sec, then slave robot contacts with the hard environment between 18 to 38 sec, consequently free motion occurs again between 38 to 50 sec. It means that a hard environment has been put in 1.5 rad. It is observed that during free motion, the slave robot follows the master one properly.

To evaluate sliding-mode impedance controller, different parameter uncertainties has been considered in the mentioned mass and even the viscous coefficient of the slave robot. Simulations are performed for overestimated and underestimated cases along with nominal one as:

<table>
<thead>
<tr>
<th></th>
<th>Mass</th>
<th>Viscous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overestimated</td>
<td>( \hat{m}_s = 3m_s )</td>
<td>( \hat{c}_s = 3c_s )</td>
</tr>
<tr>
<td>Underestimated</td>
<td>( \hat{m}_s = 0.5m_s )</td>
<td>( \hat{c}_s = 0.5c_s )</td>
</tr>
<tr>
<td>Nominal</td>
<td>( \hat{m}_s = m_s )</td>
<td>( \hat{c}_s = c_s )</td>
</tr>
</tbody>
</table>

It can be observed that the closed-loop linear teleoperation in the absence of sliding-mode based control strategy is illustrated in following figures.

**Table. 2.** The coefficients of the mass and viscous for the slave.

**Fig. 4.** Slave position in the presence of nominal impedance control.

**Fig. 5.** Slave position with the sliding mode based impedance control.

In the Figs. 4 and 5, the associated results with impedance control and the sliding-mode based impedance control strategy have been presented, respectively. Slave outputs of every case have been superposed on another one at the same Fig in order to clearly compare them.

As shown in Fig. 4, tracking performance is not achieved properly in free motion when there are uncertainties in the slave dynamic model. Note that by increasing the difference between nominal defined parameter and determined parameter in the controller, the tracking error between other cases and nominal slave trajectory is grown to a great extent. In addition, there is an unstable contact in the dynamic model in the presence of parameter uncertainties. On the other hand, the slave robot has reliable outputs for all cases by using the sliding-mode impedance control scheme. Thus, as shown in Fig. 5, it is obvious that the presented control scheme will be completely robust against the parameter uncertainties. Finally, the operation on the remote environment in the slave side is performed properly when the human in the master robot side is sent command to the slave robot.

The performance of force estimation strategy is presented in the Fig. 6.
It is shown that force estimation strategy performance is proper and it could estimate external force reliably.

5. CONCLUSION
In this research, a new robust control scheme based on the sliding-mode has been presented for the bilateral linear teleoperation system in the presence of time delay between communication channels. In the master side, a sliding-mode based impedance control scheme which can tune the master maneuverability with its desired impedance model has been designed. In the slave side, a sliding-mode based impedance control scheme has been proposed to cope with the uncertainties on the dynamic model. In addition, a force estimation strategy has been presented in order to eliminate measuring external forces directly. Consequently, employing absolute stability concept, stability analysis of the bilateral linear teleoperation including operator in the master side and remote environment has been derived. Finally, through several simulations, it has been observed the sliding-mode controller is completely robust against adverse parameter uncertainties and has reliable performance. In addition, force estimation algorithm estimates external forces properly as well.

REFERENCES