Sliding Mode Contact Force Control of n-Dof Robotics by Force Estimation

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ABSTRACT:
Control of the force exerted on an object is important for boosting system performance in robotics manipulators. Any undesired applied force may leave remarkable effects on the system, with the potential to damage the object. In addition, measuring external force is another challenge associated with such cases. Proposing an appropriate force estimation algorithm is a solution to overcome this deficiency. In this research, a control strategy is proposed to control the external force applied on the n-dof robotics. To eliminate force measurement in the controller, a force estimation strategy based on a disturbance observer is employed. Subsequently, a sliding-mode based control is implemented to cope with the force estimation error. The closed-loop stability of the system in the presence of estimated force is analytically considered. The proposed algorithm was implemented on piezoelectric actuators as the experimental setup. The experimental results confirm that by employing the proposed control scheme, precise force control is achievable. The force estimation algorithm can also suitably estimate external force.

KEYWORDS: Robotic Systems, Force Control, Sliding Mode Control, Force Estimation.

1. INTRODUCTION
The capability to manipulate the physical collision between a robotic device and environment is a basic necessity to do manipulation task properly. Unavailable error associated with modeling and presence of parameter uncertainties likely cause an increase of the contact external force and pure motion control would not be adequate for the system, eventually it causes an unstable behavior when the interaction is occurring, especially in the collision with the rigid environments.

2. LITERATURE REVIEW
Force reflection and force control becomes set to obtain a robust and flexible behavior of a robotic manipulator system in poorly structured external environments as well as safe and dependable applications in the presence of operators [1-4]. Several control structures have been suggested for free positioning as open as well as closed-loop approaches. Impedance control schemes, sliding mode as well as robust control which have been coupled with adaptive strategies have been used in such cases [5-9]. The significant drawback of the above presented control schemes is the elimination of any contact force. External forces can affect the control performance in applications such as micro assembly and cell characterization [10]. In addition, any undesired applied forces can also degrade efficiency or seriously damage the object. Subsequently, an efficient force control scheme can be developed to control the exerted external force.

A well-known force control approach was proposed considering a known dynamic model and impedance parameters of the environment [11]. The necessity for an exact and correct dynamic model of the external environment with its recognized impedance parameters is a really conservative assumption in the proposed approach. An alternative force control scheme is the hybrid position/force control [12]. In this method, the desired impedance is induced to the dynamic system to guarantee external force control. However, these proposed approaches require external force measurement. The use of force sensors poses a serious restriction on such applications. Alternatively, an accurate external force estimation approach could eliminate the necessity to measure force.

An unknown input observer was utilized as an
external force estimator for a manipulator [13]. Nevertheless, the proposed observer conditions were too restricted. A new force estimation strategy to estimate the effects of system uncertainties have also been developed [14], [15]. The above presented approaches do not consider any error dynamic for the designed observers. Therefore, the stability of the closed-loop system by using estimated external force could not be proved analytically. To solve this problem, a sliding-mode based disturbance observer in the presence of the dynamic error was investigated [16]. Nonetheless, observer sliding nature could degrade the observer performance. In other investigations, simultaneous force estimation and control approaches have also been designed. A sliding mode control scheme with external force estimation was proposed for robotics manipulators [17]. The switching behavior of the controller may reduce the controller’s efficiency. Observer-based force control was proposed as another alternative [18]. However, the stability analysis of the designed controllers in the presence of an estimated state has not been investigated.

In [19-21], external force was estimated correctly and imported into control strategy, however in [19], a velocity/force observer was proposed which is based on the Generalized Proportional Integral (GPI) method. In this research, joint velocities as well as contact forces are estimated just employing position measurements and then utilized in a force/position control scheme, but it was not proved that force control has been done analytically. In [20], an online stiffness estimation technique for robotic tasks based only on force was proposed. It could identify the stiffness of environment to some extent employing this technique, but force control was not demonstrated clearly. [21] described the interaction torque control of the Rehab-Exos, in addition, an upper-limb robotic exoskeleton with direct torque joint sensors for interaction in Virtual Environments and rehabilitation. The presented control scheme included a centralized torque control as well as separated optimal torque observers for every joint of the exoskeleton data, therefore, not requiring contact position information. The centralized torque control scheme is usually based on a full dynamics model of the presented exoskeleton, it correctly calculates the kinematics and dynamics of the robotic system and estimates the feed-forward contribution in order to compensate the dynamic loads measured using joint torque sensors, however it has not been demonstrated that force control occurred properly.

In the current work, an inner-outer external force control approach is designed. Considering the limitation caused by the existence of force sensors in such robotics tasks, an appropriate force observer is proposed. A challenging issue is force estimation error; as a result, a force control structure which is based on the sliding-mode control scheme which properly guarantees robustness to external force estimation error is derived to satisfy the desired force exertion. The stability of the whole system in the presence of estimated force is analytically achieved. A piezoelectric actuator is modeled and considered as an experimental setup. The experimental results demonstrate that the external force is controlled properly employing proposed force control algorithm. In addition, estimated force tracks the real one with appropriate accuracy.

3. MATERIAL AND METHOD

In this section, the system dynamic model of robotic system is defined. Then, a force control strategy for robotic system is proposed. A force estimation approach is proposed to cope with direct measuring of external forces. Consequently, proposed control strategy is implemented on the experimental setup.

3.1. General Nonlinear Dynamics Modeling

A general n degrees of freedom nonlinear dynamic system in the collision with the environment would be as:

\[ M(X)\ddot{x} + C(X, \dot{x})\dot{x} + G(X) = u - F_e \]  

(1)

Where, \( X = [x_1, x_2, \ldots, x_n]^T \) is the \( n \) generalized coordinates vector; \( M(X) \) shows the symmetric as well as positive-definite inertia matrix; \( C(X, \dot{x}) \) and \( G(X) \) are the Coriolis and centrifugal matrices, respectively; and \( u \) and \( F_e \) are the input control and external force, respectively.

3.2. Inner-Outer Loop Control Design

The main goal is to control external force accurately in terms of absent external force sensors. Basically, the external force should be precisely observed and then properly controlled. In addition, a conservative assumption related to the known external environment should be released.

In this section, an inner-outer loop control approach is designed for appropriate external force control of a general nonlinear dynamic system (1). Taking into account the nonlinear behavior of general mechanical systems, the inner loop controller (2) is proposed to make linear system.

\[ u = C(X, \dot{X})X + G(X) + F_e + M(X)u_1 \]  

(2)

Therefore, the system dynamic is transformed into decoupled double integrators (3) as follows.

\[ \dot{X} = u_1 \]  

(3)
In such a case, the dynamic of every individual degree of freedom could be represented as (4).

\[ \ddot{x}_i = u_{ii} \]  

(4)

In order to simplify the control design, index \( i \) is removed for further equations.

Control input \( u_i \) should be designed in such a way that the desired system impedance is achievable. Accordingly, the external force control can be implemented.

**Theorem 1**: By assuming the external force measurement, the outer loop control input (5) is achieved to produce the desired impedance for the system.

\[ u_i = \frac{1}{M_e}[-C_c \dot{x} + F_d - F_e] \]  

(5)

The closed-loop system dynamic assures system external force control that is in collision with an external unknown environment.

**Proof**: the closed-loop dynamics would be as follows by substituting the control algorithm (5) into the dynamic system (4):

\[ M_c \ddot{x} + C_c \dot{x} = F_d - F_e \]  

(6)

Therefore, the desired impedance \( Z_r(s) \) (7) is achieved for the dynamic system.

\[ Z_r(s) \ddot{x} = F_d - F_e \]  

\[ Z_r(s) = M_c s + C_c \]  

(7)

Where, \( s \) and \( Z_r(s) \) show the Laplace variable and desired system impedance, respectively. As a necessary condition, the desired proposed impedance should not include any gravity-like impedance. The equivalent electrical description associated with the closed-loop system could be observed in Fig. 1.

\[ \text{Fig.1. Force control system in equivalent electrical description.} \]

\( Z_e(s) \) shows the environment impedance. In addition, a conventional mass-damper-spring dynamic (8) is considered for the dynamic model of environment. Consequently, impedances would be absolutely unknown.

\[ F_e = Z_e(s) \dot{x} \]  

\[ Z_e(s) = m_c s + c_c + \frac{k_e}{s} \]  

(8)

Based on Fig. 1, the force relation is resulted as follows:

\[ F_e(s) \]  

\[ F_d(s) = \frac{Z_e(s)}{Z_r(s) + Z_e(s)} \]  

(9)

As a result, the error function is obtained as:

\[ E(s) = \frac{Z_r(s)}{Z_r(s) + Z_e(s)} \]  

(10)

Taking into account the defined impedance of the closed-loop system and the environment, the steady state error is eliminated (11) and the external force control is occurred.

\[ e_{ss} = \lim_{s \to 0} sE(s) = \frac{Z_r(0)}{Z_r(0) + Z_e(0)} = 0 \]  

(11)

\[ \rightarrow F_e \rightarrow F_d \]

The designed control approach can guarantee proper force control. However, the main limitation is the external force measurement. The placement of force sensors is not possible in many applications. Their high cost and probably noisy output may restrict their use as well. Consequently, a force estimation algorithm might be applicable to solve the issue.

**3.3. External Force Estimation**

With regards to general nonlinear dynamic systems (1), the proposed observer [22] is utilized as an external force estimation observer. The structure is represented as:

\[ \dot{\hat{F}}_e = -LF_e + L[u - (M(X) \ddot{X} + C(X, \dot{X}) \dot{X} + G(X))] \]  

(12)

Where, \( \dot{\hat{F}}_e \) and \( L \) are defined as estimated external force and the diagonal positive definite gain matrix, respectively:

\[ \lambda_{\text{min}}(L) \leq L \leq \lambda_{\text{max}}(L) \]  

(13)

\( \lambda_{\text{min}}(.) \) and \( \lambda_{\text{max}}(.) \) are defined as the minimum and maximum Eigenvalues of the presented gain matrix.
The stability of the proposed method should be investigated. The closed-loop dynamic of the estimator can be denoted as:

\[ \dot{\hat{F}}_e + L\hat{F}_e = \hat{F}_e \]  \hspace{1cm}  \text{(14)}

\[ \dot{\hat{F}}_e = F_e - \hat{F}_e \]

By using the positive definite Lyapunov function \( V = \frac{1}{2} F_e^T \dot{F}_e \), the time derivative is:

\[ \dot{V} = \dot{\hat{F}}_e^T \hat{F}_e = -\hat{F}_e^T L \dot{F}_e + \dot{\hat{F}}_e^T \dot{F}_e \]  \hspace{1cm}  \text{(15)}

In this condition, it has been assumed that the rate of external forces is remarkably bounded because of the lack of present information on the rate of external force, such that:

\[ 3\delta_m > 0 \quad \text{such that} \quad \| \dot{\hat{F}}_e \| < \delta, \forall t > 0 \]

Therefore:

\[ \dot{V} \leq -\lambda_{\text{min}}(L) \| \dot{\hat{F}}_e \|^2 + \delta \| \dot{\hat{F}}_e \|_2^2 \]

Where, \( \theta \in (0,1) \). Therefore, the following is concluded:

\[ \dot{V} \leq -\lambda_{\text{min}}(L)(1 - \theta) \| \dot{\hat{F}}_e \|^2 \quad \forall \| \dot{\hat{F}}_e \|_2 \geq \delta \]  \hspace{1cm}  \text{(18)}

Based on the Definition 4.2 in [23] and since the presented Lyapunov function \( (V) \) is differentiable continuously in addition to positive-definite and defined radially unbounded, therefore there are class \( \mathcal{K}_\infty \) functions \( \alpha_1(.) \) and \( \alpha_2(.) \) such that \( \alpha_1(\dot{\hat{F}}_e) \leq V(\dot{\hat{F}}_e) \leq \alpha_2(\dot{\hat{F}}_e) \). Employing Theorem 4.18 in [23], it is proved that the tracking error would be globally uniformly ultimately bounded regarding the ultimate bound specified by \( \alpha_1^{-1}(\alpha_2(\delta_m/\lambda_{\text{min}}(L)\theta)) \). Consequently, there exists \( T > 0 \), therefore the following equation occurs:

\[ |\dot{\hat{F}}_e| \leq \alpha_1^{-1}\left(\alpha_2\left(\frac{\delta_m}{\lambda_{\text{min}}(L)\theta}\right)\right) \]  \hspace{1cm}  \text{(19)}

for \( \forall \dot{\hat{F}}_e(0) \) and \( \forall t \geq T \).

\textbf{Remark 1:} The proposed observer strategy contains acceleration terms and could degrade the controller’s performance. Thus, a new auxiliary variable \( z \) is stated as \( z = \hat{F}_e - p \).

Therefore, the modified observer structure is derived as follows:

\[ \dot{z} = -Lz + L\left[u + p - \left(C(X,X)\dot{X} + G(X)\right)\right] \]  \hspace{1cm}  \text{(20)}

\[ z = \hat{F}_e - p \]

\[ p = -LM(X)\dot{X} \]

\textbf{3.4. Sliding Mode Control Scheme for a System}

The designed control algorithm includes position and velocity signals plus estimated external force along with desired impedances. Due to the force estimation error, the control performance may deteriorate. Therefore, a robust control scheme can be employed by proposing a sliding-mode control scheme such that a precise and desired model is achieved. The control input is:

\[ u = C(X,X)\dot{X} + G(X) + \tilde{F}_e + M(X)u_1 - K\text{sgn}(s) \]  \hspace{1cm}  \text{(21)}

Where, \( \tilde{F}_e \), \( K \) and \( s \) are the estimated external force, nonlinear gain and sliding surface, respectively. In addition:

\[ u_1 = \frac{1}{M_e}\left[-C_e\dot{x} + F_d - \tilde{F}_e\right] \]  \hspace{1cm}  \text{(22)}

The closed-loop system dynamic is achieved by substituting (21) in (1) and having \( \tilde{F}_e = F_e - \hat{F}_e \):

\[ M(X)\ddot{X} = \tilde{F}_e + M(X)u_1 - K\text{sgn}(s) \]  \hspace{1cm}  \text{(23)}

Now the proper sliding surface is introduced as:

\[ s = \int_{t_0}^{t} I(t)dt \]  \hspace{1cm}  \text{(24)}

Where, \( I \) is defined as:

\[ I = M(X)\ddot{X} - M(X)u_1 \]  \hspace{1cm}  \text{(25)}

By rewriting the closed-loop dynamic as well as to express it in terms of \( s \) gives:

\[ \dot{s} - \tilde{F}_e + K\text{sgn}(s) = 0 \]  \hspace{1cm}  \text{(26)}

Considering the Lyapunov function as \( V = \frac{1}{2}s^T s \), the time derivative is:

\[ \dot{V} = -s^T(\tilde{F}_e - K\text{sgn}(s)) \]  \hspace{1cm}  \text{(27)}

The system desired trajectory would converge properly towards the sliding surface, if the presented sliding conditions \( s^T s \leq \eta|s| \) is satisfied. Regarding (26) and to satisfy the sliding condition, the nonlinear gain \( K \) should be:

\[ \text{Vol. 14, No. 4, December 2020} \]
\[ \begin{align*}
    K & \geq \eta + |F_e| \\
    & \quad \text{(28)}
\end{align*} \]

To demonstrate the boundedness of \(|F_e|\) in the previous section, the appropriate magnitude of \(K\) can be reached. While the system is in the desired sliding mode control scheme, the sliding surface satisfies \(\dot{s} \to 0\) [24]. As a result, the desired model is achieved when \(l = 0\):

\[ M(X)\ddot{x} = M(X)u_1 \]

**Remark 2:** to eliminate the present undesired chattering behavior during operation caused by switching-based controller, it should be transformed into the sign function (\(\text{sgn}(s)\)) because of its continuous form as a saturation function (\(\text{sat}(\dot{\phi})\)), where \(\phi\) is the boundary layer thickness. It could be derived that the steady state error dynamic is bounded by \(\phi\). Thus, the controller structure would be:

\[ u = C(X, \ddot{x})\dot{x} + G(X) + F_e + M(X)u_1 - K\text{sat}(\dot{\phi}) \]

**Remark 3:** Noted that by deriving \(\dot{s}_1(t)\), it is required to measure acceleration signal directly. To avoid direct measuring of the acceleration signal, it is presented an auxiliary variable as follows:

\[ \dot{q} = s(t) - r(\dot{x}) \]

\[ \frac{dr(\dot{x})}{dt} = M\ddot{x} \]

Thus, it is not necessary to measure acceleration signal directly in the modified sliding surface:

\[ \dot{q} = -Mu_1 \]

\[ s = q + r(\dot{x}) \]

**Theorem 2:** The modified control algorithms (21) and (22) could guarantee external force control.

**Proof:** By substituting (21) and (22), the final closed-loop system would be as follows:

\[ M_c\ddot{x} + C_c\dot{x} = F_d - \hat{F}_e \]

Regarding the observer structure, any estimated external force could be defined in the s-domain as:

\[ \hat{F}_e = \frac{LF_e}{s + L} \]

Employing the s-domain description of estimated force gives the closed-loop dynamic model in the s-domain as:

\[ M_c\ddot{x} + C_c\dot{x} = Z_r(s)\dot{x} = F_d(s) - \frac{LF_e(s)}{s + L} \]

The closed-loop dynamic model can be rearranged in the following form:

\[ Z_r(s)\ddot{x} = F_d(s) - F_e'(s) \]

\[ F_e'(s) = \frac{LF_e(s)}{s + L} \]

Essentially, the desired impedance of the system contacts the new environment impedances as \(Z_e(s)\), which is defined as:

\[ F_e' = Z_e'(s)\dot{x} \]

\[ Z_e(s) = F(s)Z_e(s) \]

\[ F(s) = \frac{L}{s + L} \]

The equivalent electrical description of the closed-loop system is:

![Equivalent electrical diagram](image)

**Fig. 2.** Equivalent electrical description of the observer-based force control.

Regarding the equivalent electrical model and impedance of the new external force, the convergence of steady-state error could be:

\[ E(s) = \frac{Z_r(s)}{Z_r(s) + Z_e(s)} \]

\[ e_{ss} = \lim_{s \to 0} sE(s) = \frac{Z_r(0)}{Z_r(0) + Z_e(0)} = 0 \]

Finally, the proposed control scheme is as follows:
4. EXPERIMENTAL RESULTS

The designed control scheme was experimentally implemented, and the utilized setup has been presented in Fig. 4. The mentioned setup embraces a P-615 NanoCube piezoelectric actuator with maximum displacement 420 μm in the X and Y directions. To capture required data, a DS1104 dSPACE data acquisition and a controller board have been used. Matlab/Simulink software has been applied to implement control strategy. Note that dynamic model of utilized piezoelectric actuator has been presented in [25].

4.1. Control Performance Analysis

The proposed control structure was run with the experimental setup. By adjusting the controller parameters, the actuator’s external force control was investigated for \( m = 0.005, C = 900 \) (Fig 5).

It is clear that the estimated force tracks the desired external force properly. To verify the force estimation process, Fig. 6 depicts the force observation results.

Similarly, the controller performance is evaluated as displayed in Fig. 7 by another tuning as \( m = 0.004, C = 1200, m = 0.006, C = 1200 \).
Fig. 8. (a) External Force Control for \(m=0.006, C=1200\), (b) External Force Estimation for \(m=0.006, C=1200\).

It can be claimed that the estimated force is able to control environment force accurately.

Fig. 9. (a) External Force Control for \(m=0.006, C=900\), (b) External Force Estimation for \(m=0.006, C=900\).

In Fig. 9, it is shown that proposed force control algorithm is proper for time varying force as well. In addition, it is observed that estimated force converges to external force appropriately.

4.2. Force Control Analysis by Closed-Loop Gains

Closed-loop system performance is analyzed in this section. The main parameter is damping gain of the closed-loop force controller. As previously mentioned, the closed-loop force control can be assumed to be a system with position state. As a result, there are two poles: one at the origin and another depending on the damping gain. Changing damping with the following equation could effectively change the position of closed-loop poles.

\[
\frac{1}{s(M_cs + C_c)}
\]  

Obviously, increasing the gain brings the dominate pole closer to the origin. As a result, the system’s behavior would strongly depend on the first pole, causing a slower response.

The force control process is shown in Fig. 10 for different damping gains. The results confirm the previous analysis as well.

Fig. 10. Simulated External Force Control for different Gains.

In investigating the above fact, controller performance is reviewed for two different gains. The external force behaviors for these gains are compared in Fig. 11.

Fig. 11. External Force Response.
As mentioned before, increasing the damping gain could effectively decrease response time. The reduction is evident in estimated force gain, as seen in Fig. 12.

5. CONCLUSION

In this research, a sliding mode control approach was designed with a force estimation strategy to control external force exerted during a robotic manipulation. The force estimation approach was designed for time-variant external force. It was then proved that the external force estimation error was well bounded. Due to the force estimation error, a sliding-mode based control approach was proposed to satisfy the correct desired model. The inner loop control liberalizes the general nonlinear dynamic system and the outer loop control induces the desired impedance to achieve proper control by the estimated external force. An analytical investigation and the experimental results signify that decreasing the closed-loop damping impedance could increase the external force convergence to the desired value. Consequently, the experimental results verify the precise estimated signal’s external force control performance and they demonstrate that the desired external forces are controlled properly either the desired forces are constant or time-variant.

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