Event-based Controller Design for Networked Control Systems with Time-varying Random Delays

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ABSTRACT: This paper is concerned with a controller design method for Networked Control Systems (NCSs) with time-varying random delays. The proposed controller is an event-based controller and is able to effectively save the network bandwidth and energy resources of the system in comparison with common control schemes while guaranteeing the stability of the system. The proposed controller switches between two main triggering schemes: event-triggered control scheme and self-triggered control scheme. The stability issue in this combinatorial method is also considered. The validity of the proposed algorithm is confirmed via simulation results and the results are well compared with results from exerting event-triggered and self-triggered control individually.

KEYWORDS: Networked Control System, Random Delay, Event-Triggered Control, Self-Triggered Control, Bandwidth.

1. INTRODUCTION

Combining control theory and communication theory has led to introduction of networked control systems, in which a communication network is used for data transmission between sensors, controllers and plants. Transmitting data via communication network has many benefits such as reduction of wiring costs, high flexibility in order to add or remove new elements and ease of fault detection. Growing interest in NCSs in recent years, have been the outgrowth of these benefits [1-8].

Using communication network has also brought new challenges such as packet dropout, network-induced delay and limitation of network bandwidth. Packet dropout which can cause the degradation of the performance and even instability of the system, has been studied in many researches [9-13]. The effect of network-induced delay, methods of modelling and compensating different forms of delay has been the subject of many papers [14]-[18]. The stability issue and investigating the existence of a stabilizing controller or designing a stabilizing controller have been other considerable parts of NCS-related papers [19-22].

Since the negative effect of delay and packet dropout can affect the network during transmission of every packet of data; confining transferred data only to those data which are necessary for stabilizing the system is an absolute solution for reducing the instability situations. This solution also leads to reduction of network bandwidth usage and saving the energy resources of the system.

In the triggering point of view, there are three main different categories: time-triggered control scheme, event-triggered control scheme and self-triggered control scheme.

Time triggered control scheme is the first triggering method used in networked control systems. In this method, system states are sampled and sent to the controller through network periodically. Therefore the network bandwidth is always occupied with periodic data transmission. For solving this problem aperiodic control schemes were proposed, in which data is sent only if a predetermined event happens; not in constant intervals. These event-based control schemes reduce the challenges of the presence of the network but also brings new theoretical and practical issues [23-30].

In these two event-based control schemes, a triggering mechanism determines when the last sampled data must be transferred to the controller through network. In event-triggered control scheme, the data is sampled periodically and if the triggering condition is violated, the last sampled data is sent to the controller in order to generate new control signal for
stabilizing the system, otherwise the last sampled data is ignored. In self-triggered control scheme; based on our knowledge about plant dynamics; a cost function is defined, which determines the next sampling instant priority.

In [31] a model-free control method is proposed for NCSs with time-delay. A new structure for model-free control is given using Smith predictor for delay part of the problem. In [32], an event-triggered control scheme is presented based on systems output. Reference [33] is concerned with the problem of adaptive neural network tracking control for a class of uncertain switched nonlinear systems with time delays.

In [34], event-triggered method based on state-feedback is developed so that in presence of noise and when the state variable is not measurable, the method can be used.

Reference [35] is concerned with the synchronization of control problem for a class of discrete time-delay complex dynamical networks under a dynamic event-triggered mechanism.

In [36] a new method for modelling and analyzing interaction between control loops and access to communication network in first-order systems is presented. In [37], sensor is not informed of maximum delay, so creating time tags on sent packets helps the controller to know the delay for each data packet.

The benefits and drawbacks of all three control schemes are summarized in Table 1. The drawbacks of one strategy can be covered with benefits of the other strategy. Therefore in this paper a hybrid controller is proposed which switches between two event-based control schemes: event-triggered control and self-triggered control. Since this switching strategy is for the purpose of less monitoring and transmitting data in unnecessary instants, the result is less occurrence of network-induced phenomena.

Network-induced delay is also a big challenge in NCSs. In different papers, different models are assumed for different forms of delay. The most complex situations happen when there is random delay in the system.

In fact, main contribution of this paper is introducing a hybrid method which results into reducing network bandwidth usage and waste of system’s energy resources. The main problem in switching approaches is stability, which is considered and resolved in this paper in particular.

Therefore, this paper is organized as follows: in section 2 the system model and preliminaries are introduced. In section 3 the function of the proposed hybrid controller is explained. Simulation results are reported and discussed in section 4, and finally conclusion is represented in section 5.

### Table 1. Benefits and drawbacks of each control scheme.

<table>
<thead>
<tr>
<th>Control scheme</th>
<th>Weak points</th>
</tr>
</thead>
</table>
| Time-triggered | - Periodic data sampling  
|                | - Continuous effort for data transmission via network  
|                | without taking network traffic into account  
|                | - Wasting network resources for unnecessary data sampling and transmission |
| Event-triggered | - Periodic data sampling  
|                | - Waste of network resources |
| Self-triggered | - No reaction against external disturbances between two consecutive sampling instant |

### 2. SYSTEM MODEL AND PRELIMINARIES

Consider a linear time-invariant (LTI) system as:

\[
\dot{x}(t) = Ax(t) + Bu(t) \quad (1a)
\]

\[
x(0) = x_0 \quad (1b)
\]

In which \( x : \mathbb{R} \to \mathbb{R}^n \) is the system state with initial value \( x_0, u : \mathbb{R} \to \mathbb{R}^m \) is the control input and \( A \in \mathbb{R}^{n \times n} \) and \( B \in \mathbb{R}^{n \times m} \) are real matrices of appropriate dimensions.

An asymptotical stabilizing controller is necessary for our hybrid strategy so a symmetric positive definite matrix \( P \) is assumed such that

\[
\dot{x}(t) = (A - BB^T P)x(t) \quad (2)
\]

Will have an asymptotical stable equilibrium point. The matrix \( P \) is also considered to satisfy the \( H_\infty \) algebraic Riccati equation (ARE),

\[
0 = PA + A^T P - Q + R \quad (3)
\]

Where,

\[
Q = PPB^T P \quad (4)
\]

\[
R = I + \frac{1}{\gamma^2} PP \quad (5)
\]

For real constant \( \gamma > 0 \). The state feedback gain matrix is considered as \( K = -B^T P \) and the closed loop system matrix is denoted as \( A_s = A - BB^T P \).

Therefore, we can rewrite equation 1 as:
The sampled-data system is finite gain L2 stable with a gain less than $\gamma/\beta$ if:

$$e_k(t)Qe_k(t) < (1 - \beta^2) \|x(t)\|^2 + x_0^TQx_0$$  \hspace{1cm} (9)$$

For all $t \in [f_k, f_{k+1})$ and any $k = 0, ..., \infty$.

A weaker sufficient condition for L2 stability of the system is presented in the following corollary.

**Corollary 1:** Consider the sampled-data system in equation 8 with admissible release time and finishing time sequences. $\beta$ is a real constant in the interval $(0,1)$ and the matrix $Q$ is defined as in equation 3 such that the matrix

$$M = (1 - \beta^2)I + Q$$  \hspace{1cm} (10)$$

Has full rank and the inequality

$$e_k(t)^TMe_k(t) \leq x_0^TMx_0$$  \hspace{1cm} (11)$$

Is satisfied for $t \in [f_k, f_{k+1})$ for all $k = 0, ..., \infty$ then the sampled data system is L2 stable with a gain less than $\gamma/\beta$.

In event-triggered control scheme and self-triggered control scheme, we need an inequality in order to check if the event has happened. The inequalities 9 or 11 can be used for event-triggered strategy and by replacing $x_t$ with $e_t$, those inequalities can be used for self-triggered strategy.

The function $z_k: [r_k, f_{k+1}) \rightarrow \mathbb{R}^n$ is defined as

$$z_k(t) = \sqrt{(1 - \beta^2)I + Q}e_k(t) = \sqrt{M}e_k(t)$$  \hspace{1cm} (12)$$

And the function $\rho: \mathbb{R}^n \rightarrow \mathbb{R}$ is given by

$$\rho(x) = \sqrt{x^TMx}$$  \hspace{1cm} (13)$$

Where, $x \in \mathbb{R}^n$. If we can prove for any $\delta \in (0,1]$ that

$$\|z_k(t)\|_2 \leq \delta \rho(x_r)$$  \hspace{1cm} (14)$$

For all $t \in [f_k, f_{k+1})$ for any $k = 0, ..., \infty$, then the hypotheses in corollary 1 result into finite gain L2 stability of the sampled-data with a gain less than $\gamma/\beta$.

**Theorem 2:** It is assumed that for some $\delta \in (0,1]$, the sequence of release times $\{r_k\}_{k=0}^{\infty}$ satisfies

$$\|z_k(t)\|_2 \leq \delta \rho(x_r)$$  \hspace{1cm} (15)$$

Where, $f_k = r_k$ for all $k = 0, ..., \infty$ and the sampled-data system in equation 8 is considered.

If the release and finishing time sequences are admissible and the sampled-data system in equation 8 is considered.

If the release and finishing time sequences are admissible and the sampled-data system is L2-stable with a gain less than $\frac{\gamma}{\beta}$, then each task sampling period will satisfy
In which $\alpha$ is a real constant and $\mu_0: \mathbb{R}^n \to \mathbb{R}$ is a real-valued function as:

$$\alpha = \| \sqrt{M} A \sqrt{M}^{-1} \|$$

(17)

$$\mu_0(x_r) = \| \sqrt{M} A_d x_r \|_2$$

(18)

It is concluded from theorem 2 that if we set

$$r_{k+1} = r_k + \frac{1}{\alpha} \ln(1 + \delta \alpha \frac{\rho(x_r)}{\mu(x_r)})$$

(19)

Then we are assured that the system’s induced L2 gain is less than $\frac{\gamma}{\beta}$.

Corollary 2. Consider the sampled-data system. It is assumed that for some $k$, $r_{k-1} \leq f_{k-1} \leq r_k$. If for some $\varepsilon \in (0,1)$, the inequality

$$0 \leq D_k = f_k - r_k \leq L_1(x_r, x_r^-; \varepsilon)$$

(20)

For all $t \in [r, f]$, the $k$th trigger signal $z_k$ satisfies

$$\| z_k(t) \|_2 \leq \phi(x_r, x_r^-, t - r) \leq \varepsilon \rho(x_r)$$

(21)

For all $t \in [r, f]$,

$$L_1(x_r, x_r^-; \varepsilon) = \frac{1}{\alpha} \ln(1 + \varepsilon \alpha \frac{\rho(x_r)}{\mu(x_r)})$$

(22)

In which $\phi: \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$ is a real-valued function given by

$$\phi(x_r, x_r^-, t - r) = \frac{\mu_1(x_r, x_r^-)}{\alpha} \left( e^{\alpha(t-r)} - 1 \right)$$

(23)

And $\mu_1: \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ is a real-valued function given by

$$\mu_1(x_r, x_r^-) = \sqrt{M}(Ax_r - BB^TPx_r^-)$$

(24)

Theorem 3. Consider the sampled-data system in equation 8. For given $\varepsilon \in (0,1)$ and $\delta \in (0,1)$, we assume that the initial release and finishing times satisfy

$$r_{-1} = r_0 = f_0 = 0$$

(25)

For any non-negative integer $k$, the release times are generated by the following recursion,

$$r_{k+1} = f_k + L_2(x(r_k), x(r_{k-1}), \delta)$$

(26)

And the finishing times satisfy

$$r_{k+1} \leq f_{k+1} \leq r_{k+1} + \xi(x(r_k), \varepsilon, \delta)$$

(27)

Where $L_2: \mathbb{R}^n \times \mathbb{R}^n \times (0,1) \to \mathbb{R}$ is defined as

$$L_2(x_r, x_r^-, \eta) = \frac{1}{\alpha} \ln(1 + \alpha \frac{np(x_r) - \phi(x_r, x_r^-)}{\mu_0(x_r) + a\phi(x_r, x_r^-)})$$

(28)

And $\xi: \mathbb{R}^n \times (0,1) \to \mathbb{R}$ is defined as

$$\xi(x_r^-, \varepsilon, \delta) = \frac{1}{\alpha} \ln(1 + \varepsilon \alpha \frac{\mu_1(x_r) - \phi(x_r, x_r^-)}{a\delta \rho(x_r^-) + \mu_0(x_r^-)})$$

(29)

Then it is concluded that the sequences of release times, $\{r_k\}_{k=0}^\infty$, and finishing times, $\{f_k\}_{k=0}^\infty$, will be admissible and the sampled-data system is finite gain L2 stable with an induced gain less than $\frac{\gamma}{\beta}$.

3. PROPOSED CONTROLLER

The time diagram of the system response has two basic parts, transient part and steady state part. Since in transient time, the system states’ changes are fast, control signal must be modified continuously in order to direct variable states of the system towards stability. In contrast, during steady state interval, a proper control signal is required just in order to keep the system states near stability mode. The main idea in the proposed control strategy is using a switching approach for the applied control strategy based on the system mode. If the system is in transient mode, the event-triggered control strategy is used and when the system is in steady state mode, the self-triggered control strategy is used.

The switching action is done based on the inequality

$$\| \text{relative error} \|_2 \geq \text{threshold}$$

(30)

Where, “relative error” is $(x_r - x_r^-)/x_r$ and the threshold is the factor determining the border between transient mode and steady state mode.

At the time the switching action happens, the initial state of the new strategy is set to the last state of the previous strategy. So because of the guaranteed L2 stability of the system in both of two types of strategies from any initial condition, the L2 stability of the system after switching is also guaranteed. Thereby the stability problem is resolved in this way.

4. SIMULATION RESULTS

In this section, the proposed control strategy is applied to an inverted pendulum and the results are compared to applying only one type of control strategies.

The plant’s linearized state equation is
\[
\dot{x}(t) = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & -mg/M & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & g/l & 0
\end{bmatrix} x(t) + \\
\begin{bmatrix}
0 \\
1/M \\
0 \\
-1/(MI)
\end{bmatrix} u(t)
\]

(31)

In which \( M \) is the cart mass, \( m \) is the mass of the pendulum bob and \( l \) is the length of the pendulum arm and \( g \) is the gravitational acceleration. System state vector; which is measured in each sampling instant; is as \( x = [y \ y \ \dot{\theta} \ \dot{\theta}]^T \) where \( y \) is the cart’s position and \( \theta \) is the pendulum bob’s angle with respect to the vertical.

In this part, three different algorithms are used and the results are compared together: event-triggered control, self-triggered control and hybrid control. Since reducing the energy usage is our greatest concern in this paper, the parameter \( \Psi \) is defined as \( \Psi = \text{sent} \ast \text{sampled} \ast \text{error}_\text{norm} \). The less \( \Psi \) is, the less energy is used and the better result is obtained.

The system’s initial state is set to \( x_0 = [0.98 \ 0.2 \ 0]^T \). Solving the Riccati equation in equation 3, and using the equation 6, yields

\[
K = -B^TP = [2 \ 12 \ 378 \ 210]
\]

Fig. 1 plots the state trajectory of the closed-loop system using the above controller gain.

Fig. 2 shows the state trajectory of the closed-loop system using each of three discussed control schemes (event-triggered control, self-triggered control and hybrid control) compared to the time-triggered control scheme. For more visibility, the 5 seconds running results are included.

Fig. 3 illustrates the control signal applied to the plant in each of three control schemes. It can be seen that all the signals converge to the level of time-triggered control signal which shows the proper behavior of the system with each type of controller.

The error signal for this system is defined as \( e(t) = ||x(t) - x_c(t)||_2 \) where \( x(t) \) is the event-based system’s response and \( x_c(t) \) is the system’ response without applying any event-based control strategy. The error signals' plots are included in Fig. 4.

In order to show the performance of the proposed controller in long time running of the system; Fig. 5 illustrates the system condition in which the system mode changes between transient and steady states. As can be seen clearly, the state trajectories are so alike to continuous-time system. The error-norm is of small amplitude in comparison with system states’ amplitude. The control signal is also plotted in Fig. 5 (c).
Fig. 2. System states using (a) event-triggered controller, (b) self-triggered controller, (c) hybrid controller.

Fig. 3. Control signal using three different types of control schemes.
Table 2 shows the dependency of the $\Psi$ factor to different values of $\epsilon$ and $\beta$. For each of three algorithms, presented results in Table 3, declare the efficiency of the proposed algorithm. Since the system is initially in transient state, the number of sent packets and the $\Psi$ factor for STC is far more than ETC and the Hyb.C which shows that using STC in transient interval is an inappropriate choice. The number of sent packets in proposed algorithm is considerably less than the other two algorithms. As the length of interval increases, the relative greatness of number of sent packets in ETC to Hyb.C increases. $\tau$ parameter is the ratio of the whole running interval to number of sent packets. This parameter is interpreted as the average number of intervals between two consecutive release times. In simulation results, each interval is considered as $h=1e^{-5}$.

The values of the $\Psi$ factor for different running intervals of the algorithms, shows that this factor for the ETC is about 3 to 4 times greater than Hyb.C. With increase in running interval, the steady state interval is also included and the STC is more involved in Hyb.C. So the better performance of STC than ETC in steady state interval can be observed in this case which confirms the good structure of the proposed algorithm for relating the ETC to transient state interval and the STC to steady state interval.

Table 2. Dependency of the $\Psi$ factor to $\epsilon$ and $\beta$ parameters in different control schemes.

<table>
<thead>
<tr>
<th>Control Scheme</th>
<th>$\epsilon$</th>
<th>$\beta$</th>
<th>$\Psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event-triggered</td>
<td>0.1</td>
<td>0.5</td>
<td>8.1545e9</td>
</tr>
<tr>
<td>control</td>
<td>0.1</td>
<td>0.8</td>
<td>8.7891e9</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>0.5</td>
<td>4.5675e9</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>0.8</td>
<td>4.4307e9</td>
</tr>
<tr>
<td>Self-triggered</td>
<td>0.1</td>
<td>0.5</td>
<td>1.5253e12</td>
</tr>
<tr>
<td>control</td>
<td>0.1</td>
<td>0.8</td>
<td>9.4322e12</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>0.5</td>
<td>1.6178e12</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>0.8</td>
<td>9.5010e12</td>
</tr>
<tr>
<td>Proposed hybrid</td>
<td>0.1</td>
<td>0.5</td>
<td>4.1665e9</td>
</tr>
<tr>
<td>control</td>
<td>0.1</td>
<td>0.8</td>
<td>4.5363e9</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>0.5</td>
<td>4.1665e9</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>0.8</td>
<td>4.5363e9</td>
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</tbody>
</table>
5. CONCLUSION

In this paper a hybrid controller for networked control systems with time-varying delay is provided. The suitability of ETC for transient mode and STC for steady mode is shown via simulation results. Results also show that the weak performance of ETC in steady mode and STC in transient mode can be covered using switching approach. The results can also be obtained using any other two stabilizing ETC and STC strategies. Less energy consumption due to using proper strategy in different intervals is also another noticeable result of this paper.

REFERENCES


