Predictive Control Based Speed, Torque and Flux Prediction of a Double Stator Induction Motor

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ABSTRACT: The predictive control based speed, flux and torque prediction of a double stator induction motor is proposed in this research paper; the model of the DSIM and the direct vector control of the system have performed, subsequently, the classical PI controllers for the speed control, the flux, and thus for setting the stator’s currents have adopted. In order to minimize the transient control and to reduce the impact of measurement noise on the control signal, instead of vector control technique which requires the flux and torque estimation, the multivariable generalized predictive control is used. The results have shown the effectiveness of the proposed method, especially in the parameters variation and/or the change of the reference speed.

KEYWORDS: Double Stator Induction Motor, Direct Field Oriented Control, Predictive Control.

1. INTRODUCTION
The dual star induction machine (DSIM) or double stator induction machine has been the subject of numerous studies for a long time. It has the advantage of being robust, reliable and poorly functioning in degraded operation. Unfortunately, it has a major disadvantage, its dynamic structure is highly nonlinear and the internal variables such as the electromagnetic torque and the rotor flux are strongly coupled, which makes the control of this machine complex [1-3].

Indeed, the vector control makes it possible to envisage a decoupling between the electromagnetic torque and the rotor flux of the machine and to achieve a control comparable to that of the dc machines [4]. This method of control was carried out in 1971 and 1972 by HASS and BLASCHKE [5]. Several similar techniques, but with some differences in the interpretation of key concepts, have been proposed in the field of induction machine control [6], [7]. For example, scalar control, direct torque control, input/output linearization control, etc. Most of these techniques are based on linear modeling of the machine. For the reason of facilitation processing, the case of a linear model also has the advantage of being situated in a very rich theoretical context [8].

The generalized predictive control proves a sufficiently complete structure proposed to solve a very general problem, providing a stable system for a set of adjustment parameters [9], [10]. This strategy allows to control processes with non-minimal phase shift, with unstable or poorly damped poles, with dead times, constant or unknown, or with an unknown order model. It has proven its efficiency, flexibility and success in industrial applications [11].

The main contribution of this paper is the implementation of a high-performance predictive control law for a double stator induction motor, with as objectives: improve the pursuit of trajectories, guarantee the stability, the robustness to the variations of the parameters and disturbance rejection. This control strategy is used to develop a multivariable predictive control technique based on the prediction of the electromagnetic torque and the rotor flux to generate the switching states of the inverter and consequently the supply voltages of the DSIM.

The paper is organized as follows: the double stator induction motor (DSIM) model and the vector control strategy are presented in section 2 and 3 respectively. In Section 4 the multivariable predictive control strategies are discussed. In section 5 and 6 the application of the resulting predictive control of DSIM is provided. Finally, the overall proposed predictive control scheme of DSIM shown in Fig.1 is used for numerical simulation and the related results and remarks are presented.
2. DSIM MODEL

The Orthogonal subspaces ρ-β model of the double stator induction machine is presented in Fig 2. The equations of this machine can be expressed in (α, β) axes where the attributed reference is the stator field [3, 4].

1.1. Voltages Equations

By choosing a referential related to the stator field, we obtain the following system of equations [1-4]:

\[
\begin{align*}
\Phi_{s1\alpha} &= L_{s1\alpha}i_{s1\alpha} + L_{m}\left(i_{s\alpha} + i_{s\beta} + i_{ra}\right) \\
\Phi_{s1\beta} &= L_{s1\beta}i_{s1\beta} + L_{m}\left(i_{s\beta} + i_{s\beta} + i_{ra}\right) \\
\Phi_{s2\alpha} &= L_{s2\alpha}i_{s2\alpha} + L_{m}(i_{s\alpha} + i_{s\alpha} + i_{ra}) \\
\Phi_{s2\beta} &= L_{s2\beta}i_{s2\beta} + L_{m}(i_{s\beta} + i_{s\beta} + i_{ra}) \\
\Phi_{ra} &= L_{r}i_{ra} + L_{m}(i_{s\alpha} + i_{s\alpha} + i_{ra}) \\
\Phi_{rb} &= L_{r}i_{rb} + L_{m}(i_{s\beta} + i_{s\beta} + i_{ra})
\end{align*}
\]

(2)

Where:
- \(L_{s1\alpha}\) : First and second stator inductance in stationary frame
- \(i_{s1\alpha}, i_{s2\alpha}\) : First and second stator currents in the stationary frame
- \(\Phi_{s1\alpha}, \Phi_{s2\alpha}\) : First and second stator flux in the stationary frame
- \(\Phi_{s1\beta}, \Phi_{s2\beta}\) : Rotor flux in stationary frame
- \(\omega_r\) : Rotor frequency
- \(R_{s1\alpha}, R_r\) : First and second stator and rotor resistance

Repeating the system of equations (2) in (1), we obtain the mathematical DSIM model (3):

\[
\begin{align*}
v_{s1\alpha} &= R_{s1\alpha}i_{s1\alpha} + \frac{d\Phi_{s1\alpha}}{dt} + \frac{L_{s1\alpha}L_{r}}{L_{s1\alpha} + L_{r}} \frac{di_{s1\alpha}}{dt} + \frac{L_{m}}{L_{s1\alpha} + L_{r}} \frac{d\Phi_{ra}}{dt} + \frac{L_{m}}{L_{s1\alpha} + L_{r}} \frac{d\Phi_{rb}}{dt} \\
v_{s1\beta} &= R_{s1\beta}i_{s1\beta} + \frac{d\Phi_{s1\beta}}{dt} + \frac{L_{s1\beta}L_{r}}{L_{s1\beta} + L_{r}} \frac{di_{s1\beta}}{dt} + \frac{L_{m}}{L_{s1\beta} + L_{r}} \frac{d\Phi_{ra}}{dt} + \frac{L_{m}}{L_{s1\beta} + L_{r}} \frac{d\Phi_{rb}}{dt} \\
v_{s2\alpha} &= R_{s2\alpha}i_{s2\alpha} + \frac{d\Phi_{s2\alpha}}{dt} + \frac{L_{s2\alpha}L_{r}}{L_{s2\alpha} + L_{r}} \frac{di_{s2\alpha}}{dt} + \frac{L_{m}}{L_{s2\alpha} + L_{r}} \frac{d\Phi_{ra}}{dt} + \frac{L_{m}}{L_{s2\alpha} + L_{r}} \frac{d\Phi_{rb}}{dt} \\
v_{s2\beta} &= R_{s2\beta}i_{s2\beta} + \frac{d\Phi_{s2\beta}}{dt} + \frac{L_{s2\beta}L_{r}}{L_{s2\beta} + L_{r}} \frac{di_{s2\beta}}{dt} + \frac{L_{m}}{L_{s2\beta} + L_{r}} \frac{d\Phi_{ra}}{dt} + \frac{L_{m}}{L_{s2\beta} + L_{r}} \frac{d\Phi_{rb}}{dt}
\end{align*}
\]

(3)

Where:
- \(L_{s1\alpha}\) : First and second stator inductance
- \(L_r\) : Rotor inductance
- \(L_m\) : Mutual inductance

1.2. Flux Equations

The relations between flux and currents are given by [1-4]:

\[
\begin{align*}
0 &= -\frac{L_{s1\alpha}}{T_r}(i_{s\alpha} + i_{s\alpha}) + \frac{1}{T_r} \Phi_{ra} + \frac{d\Phi_{ra}}{dt} + \omega_r \Phi_{rb} \\
0 &= -\frac{L_{m}}{T_r}(i_{s\beta} + i_{s\beta}) + \frac{1}{T_r} \Phi_{rb} + \frac{d\Phi_{rb}}{dt} + \omega_r \Phi_{ra}
\end{align*}
\]
**1.3. Mechanical Equations**

The equation of the electromagnetic torque is [4], [12]:

\[
T_{em} = p \frac{L_m}{L_m + L_r} \left[ (i_{\alpha 1} + i_{\beta 2}) \Phi_{ra} - (i_{\alpha 2} + i_{\beta 2}) \Phi_{r\beta} \right] \tag{4}
\]

Where:
- \( T_{em} \) : Electromagnetic torque;
- \( p \) : Number of pole pairs

The mechanical equation is:

\[
J \frac{d\Omega_r}{dt} = T_{em} - T_L - k_f \Omega_r \tag{5}
\]

Where:
- \( J \) : Inertia;
- \( \Omega_r \) : Mechanical rotor speed
- \( T_L \) : Load torque
- \( k_f \) : Viscous friction coefficient

**3. DIRECT FIELD ORIENTED CONTROL OF DSIM**

Fig. 3 presents the main blocks of the direct field-oriented control of a double stator induction machine [8].

**3.1. Rotor Flux Amplitude and Position Estimation of DSIM**

In the direct vector control, knowledge of the rotor flux (amplitude and phase) is required to ensure the decoupling between the torque and flux. Indeed, the position of the rotor flux \( \theta_s \) is calculated algebraically from the information on the rotor flux [13], [14].

\[
\begin{align*}
\dot{\Phi}_r &= \sqrt{\Phi_{ro}^2 + \Phi_{r\beta}^2} \\
\dot{\theta}_s &= \arctan \left( \frac{\Phi_{r\beta}}{\Phi_{ro}} \right)
\end{align*}
\tag{6}
\]
These components can be expressed from the DSIM voltage model; equation (3):

\[
\begin{align*}
\Phi_{r\alpha} &= \frac{L_s + L_m}{L_m} \left[ V_{s\alpha 1} - R_{s\alpha 1} - \sigma(L_{s} + L_m) \frac{di_{s\alpha 1}}{dt} \right] - \frac{L_m L_s}{L_m + L_r} \frac{di_{s\beta 1}}{dt} \\
\Phi_{r\beta} &= \frac{L_s + L_m}{L_m} \left[ V_{s\beta 1} - R_{s\beta 1} - \sigma(L_{s} + L_m) \frac{di_{s\beta 1}}{dt} \right] - \frac{L_m L_s}{L_m + L_r} \frac{di_{s\beta 2}}{dt}
\end{align*}
\]

(7)

3.2. The Rotor Flux Orientation

The principle of the orientation shown in Fig. 4 aligns the rotor flux on the direct axis of Park’s axes [4], [12]. Thus, we obtain the orientation of the rotor flux:

\[
\begin{align*}
\Phi_r &= \Phi_{rd} \\
\Phi_{rq} &= 0
\end{align*}
\]

(8)

The following equations of rotor flux and electromagnetic torque are used:

\[
\begin{align*}
\frac{d\Phi^*_r}{dt} &= \frac{1}{T_r} L_m (i^*_{sd 1} + i^*_{sd 2}) - \Phi^*_r \\
T^*_{em} &= p \frac{L_m}{L_m + L_r} (i^*_{s q 1} + i^*_{s q 2}) \Phi^*_r
\end{align*}
\]

(9)

After the Laplace transform, we can write:

\[
\begin{align*}
\Phi^*_r &= \frac{L_m}{1 + T_r s} (i^*_{sd 1} + i^*_{sd 2}) \\
T^*_{em} &= p \frac{L_m}{L_m + L_r} (i^*_{s q 1} + i^*_{s q 2}) \Phi^*_r
\end{align*}
\]

(10)

The two stator windings are identical, so the powers provided by this two windings system are the same, hence:

\[
\begin{align*}
i^*_{sd 1} &= i^*_{sd 2} = \frac{1 + T_r s}{2L_m} \Phi^*_r \\
i^*_{s q 1} &= i^*_{s q 2} = p \frac{L_m + L_r}{2L_m} \Phi^*_r T^*_{em}
\end{align*}
\]

(11)

Fig. 3. The direct field-oriented control scheme of DSIM.
3.3. The Compensation Method

The compensation method is concerned with the regulation of four currents loops while neglecting the coupling terms [13].

Four new independent voltages $V_{ds1}$, $V_{qs1}$, $V_{ds2}$, and $V_{qs2}$ are introduced to the decoupling such that:

$$
\begin{align*}
V_{ds1}^* &= V_{ds1}^c + V_{ds1}^r \\
V_{qs1}^* &= V_{qs1}^c + V_{qs1}^r \\
V_{ds2}^* &= V_{ds2}^c + V_{ds2}^r \\
V_{qs2}^* &= V_{qs2}^c + V_{qs2}^r
\end{align*}
$$

(12)

The compensation voltages at the regulator’s output are given by [4]:

$$
\begin{align*}
V_{ds1}^c &= -\hat{\omega}_s \left[ L_{sq1} i_{sq1} + \frac{L_{sq}}{R} \omega_q \phi_r \right] \\
V_{qs1}^c &= \hat{\omega}_s \left[ L_{s1} i_{s1} + \frac{L_{s1}}{R} \phi_r \right] \\
V_{ds2}^c &= -\hat{\omega}_s \left[ L_{sq2} i_{sq2} + \frac{L_{sq}}{R} \omega_q \phi_r \right] \\
V_{qs2}^c &= \hat{\omega}_s \left[ L_{s2} i_{s2} + \frac{L_{s2}}{R} \phi_r \right]
\end{align*}
$$

(13)

The references control voltages $V_{sd1}$, $V_{sq1}$, $V_{sd2}$, and $V_{sq2}$ are reconstructed from voltages $V_{ds1}^c$, $V_{qs1}^c$, $V_{ds2}^c$, and $V_{qs2}^c$.

We define a new system to which the actions of the $d$ and $q$ axes are decoupled:
The matrix $G(z) \in \mathbb{C}^{n \times m}$ is called the transfer matrix linking the input $U(z)$ to the output $Y(z)$ [16].

![Multivariable system](image)

**Fig. 5.** The multivariable system of $m$ inputs and $n$ outputs.

### 4.2. System Matrix Determination

However, if the system is a multidimensional, then $A(q^{-1})$ and $B(q^{-1})$ are transfer matrices. Since $G(q^{-1})$ is also a transfer matrix, the polynomials $A(q^{-1})$ and $B(q^{-1})$ cannot be simply determined by taking the denominator and the numerator of $G(q^{-1})$. A simple method for the determination of $A(q^{-1})$ and $B(q^{-1})$ is presented by Camacho in [10], thus a very good detailed description could also be found in the Geering publication [17].

For simplicity reasons, only the method of Camacho [10] is treated. We have:

$$G(q^{-1}) = A(q^{-1})^{-1}B(q^{-1})q^{-1}$$  \hspace{1cm} (27)

Now the simplest is to assume that $A(q^{-1})$ is a diagonal matrix whose diagonal elements are the smallest common multiples of the denominators of the corresponding lines of $G(q^{-1})$, and the transfer matrix $B(q^{-1})$ can simply be computed with:

$$B(q^{-1}) = A(q^{-1})G(q^{-1})q$$  \hspace{1cm} (28)

### 4.3. The Multivariable CARIMA Model

Analog to the monovariable system, the following approach is taken for the CARIMA model [18]:

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) + \frac{C(q^{-1})}{\Delta} \zeta(k)$$  \hspace{1cm} (29)

Where:

$$A(q^{-1}) = I + A_1q^{-1} + A_2q^{-2} + \ldots + A_mq^{-m}$$  

$$B(q^{-1}) = B_0 + B_1q^{-1} + B_2q^{-2} + \ldots + B_nq^{-n}$$  

$$C(q^{-1}) = I + C_1q^{-1} + C_2q^{-2} + \ldots + C_nq^{-nc}$$  \hspace{1cm} (30)

The dimensions of the individual matrices are:

- $A(q^{-1})$ : $(n \times n)$ Matrix
- $B(q^{-1})$ : $(n \times m)$ Matrix
- $C(q^{-1})$ : $(n \times m)$ Matrix
- $y(k)$ : $(n \times 1)$ Vector
- $u(k)$ : $(m \times 1)$ Vector
- $\zeta(k)$ : $(n \times 1)$ Vector

With:

- $n$ : The number of system outputs.
- $m$ : The number of system inputs.

### 4.4. The J-Step Ahead Multivariable Predictor

As in SISO case, a matrix Diophantine equation is used for calculating the matrix predictor [19,20]:

$$A_m(q^{-1}).C(q^{-1}) = \Delta(q^{-1})A(q^{-1})F_j(q^{-1}) + q^{-d}G_j(q^{-1})$$  \hspace{1cm} (31)

With:

$$F_j(q^{-1}) = F_{j,0} + F_{j,1}q^{-1} + F_{j,2}q^{-2} + \ldots + F_{j,j}q^{-(j-1)}$$  

$$G_j(q^{-1}) = G_{j,0} + G_{j,1}q^{-1} + G_{j,2}q^{-2} + \ldots + G_{j,m}q^{-ma}$$

$$A_m(q^{-1}) = A_0 + A_1q^{-1} + A_2q^{-2} + \ldots + A_n q^{-ma}$$

Where:

- $F_j(q^{-1})$ : $(n \times n)$ Matrix;
- $G_j(q^{-1})$ : $(n \times n)$ Matrix;
- $A_m(q^{-1})$ : $(n \times n)$ Matrix;

After resolving equation (31) for $\Delta A(q^{-1})F_j(q^{-1})$ and after inserting the result in equation (30) which has been multiplied by $\Delta F_j(q^{-1})q^j$, the prediction equation can be obtained:

$$A_m(q^{-1}).C(q^{-1})\hat{y}(k+j) = G_j(q^{-1})y(k) + + B(q^{-1})F_j(q^{-1})\Delta u(k+j-d) + C(q^{-1})F_j(q^{-1})\zeta(k+j)$$  \hspace{1cm} (32)

All noise terms are in the future, too, which means they are not known and they can be neglected. Therefore, the best prediction is [9], [14]:

$$A_m(q^{-1}).C(q^{-1})\hat{y}(k+j) = G_j(q^{-1})y(k) + + B(q^{-1})F_j(q^{-1})\Delta u(k+j-d)$$  \hspace{1cm} (33)

And as:

$$B(q^{-1})F_j(q^{-1}) = H_j(q^{-1}).A_m(q^{-1}).C(q^{-1}) + q^{-d}J_j(q^{-1})$$  \hspace{1cm} (34)

Where, in contrary the one-dimensional case, $H_j(q^{-1})$ is a transfer matrix, the part forming the free response is completely inside the term $q^{-d}J_j(q^{-1})$. 

70
A_m(q^{-1})C(q^{-1})\hat{y}(k+j) = A_m(q^{-1})C(q^{-1})H_j(q^{-1}) \Delta u(k+j-d) \quad (35)

Thus, the prediction equation becomes:

\hat{y}(k+j) = H_j(q^{-1}) \Delta u(k+j-d) + E(k+j) \quad (36)

Where:

E(k+j) = G_j(q^{-1})y(k) + J_j(q^{-1}) \Delta u(k-1)

With:

H_j(q^{-1}), J_j(q^{-1}) : (n X m) Matrix;
E(k) : (n X 1) Vector.

Therefore, the j-step predictions for a MIMO system are:

\begin{align*}
\hat{y}(k+1) &= H_1(q^{-1}) \Delta u(k+1-d) + E(k+1)\\
\hat{y}(k+2) &= H_2(q^{-1}) \Delta u(k+2-d) + E(k+2)\\
\hat{y}(k+3) &= H_3(q^{-1}) \Delta u(k+3-d) + E(k+3)\\
&\vdots\\
\hat{y}(k+N_2) &= H_{N_2}(q^{-1}) \Delta u(k+N_2-d) + E(k+N_2)
\end{align*}

Similar to the monovariable system, the term $H_j(q^{-1}) \Delta u(k+j-d)$ of equation (36) is associated with a multiplication of real matrices $H, U, E$. So:

$Y = HU + E \quad (37)$

A quadratic cost function is applied to the prediction values of (37) to compute an optimal sequence of the control signal [9].

The cost function is the same as for the one-dimensional control [20].

$J_{GPC} = \sum_{j=0}^{N_2} \| \hat{y}(k+j) - \omega_{ref}(k+j) \|^2 + \lambda \sum_{j=1}^{N_1} \| \Delta u(k+j-d) \|^2 \quad (38)$

As in the one-dimensional case, a multi-dimensional cost function can be implemented by substituting (37) into (38):

$J_{GPC} = (HU + E - W)^T(UHU + E - W) + \lambda U^TU \quad (39)$

In order to minimize this equation, it is first developed in distinct terms [19-20]:

$J_{GPC} = U^T(H^TH + \lambda I)U + U^TH^T(E - W) + (E - W)^THU + (E - W)^T(E - W) \quad (40)$

Due to the identity:

$U^TH^T(E - W) = (E - W)^THU \quad (41)$

$J_{GPC}$ can be further simplified by:

$J_{GPC} = U^T(H^TH + \lambda I)U + 2U^TH^T(E - W) + (E - W)^T(E - W) \quad (42)$

The analytic minimization $\frac{dJ_{GPC}}{dU} = 0$ gives [21, [22]:

$\hat{U} = (H^TH + \lambda I_{N_2})^{-1}H^TW - E \quad (43)$

5. MULTIVARIABLE PREDICTION CONTROL OF DSIM

The objective of this control strategy is to control simultaneously the electromagnetic torque and the rotor flux norm. The multivariable predictive control law developed in section (5) is used to track trajectories (torque and flux norm) to generate the reference voltages (Fig. 6).

5.1. Synthesis of the MIMO Predictive Control law

The multivariable predictive control for the DSIM drive can be designed (Fig. 7).

In our case the selected outputs and inputs are:

$Y = \begin{bmatrix} \Phi_r \\ T_{em} \end{bmatrix}$, $\hat{Y} = \begin{bmatrix} \hat{\Phi}_r \\ \hat{T}_{em} \end{bmatrix}$ and $U_r = \begin{bmatrix} V_{dsr}^r \\ V_{qsr}^r \end{bmatrix}$

The prediction of the outputs is given by equation (44):

$\begin{bmatrix} \hat{\Phi}_r(k+j) \\ \hat{T}_{em}(k+j) \end{bmatrix} = G_j(q^{-1}) \begin{bmatrix} \Phi_r(k) \\ T_{em}(k) \end{bmatrix} + H_j(q^{-1}) \begin{bmatrix} \Delta V_{sd}(k+j-1) \\ \Delta V_{sq}(k+j-1) \end{bmatrix}$

$+ J_j(q^{-1}) \begin{bmatrix} \Delta V_{sd}(k-1) \\ \Delta V_{sq}(k-1) \end{bmatrix} \quad (44)$

With:

$H_j(q^{-1}), J_j(q^{-1}) : (2 X 2) Matrix; \quad G_j(q^{-1}) : (2 X 2) Matrix.$

The prediction model is assumed to be linear time invariant model (LTI), modeling the dynamic interactions between two sets of inputs $(V_{sd}, V_{sq})$ and two sets of outputs $(\Phi_r, T_{em})$.

Since from reference flux $\Phi_r$ and reference torque $T_{em}$, the reference currents $(I_{dsr}^* \text{ et } I_{qsr}^*)$ can be deduced directly from the equations of the system (45).
\[
\begin{align*}
\Phi^*_r &= \frac{2L_{em}}{1 + T_s} I^*_{ds} \\
T^*_{em} &= \frac{2pL_m}{L_m + L_r} \Phi^*_r I^*_q
\end{align*}
\] (45)

The reference voltages \( V_{ds}^r \) and \( V_{qs}^r \) are then reconstituted from the currents \( I^*_{ds} \) and \( I^*_q \) by:

\[
\begin{align*}
\Phi^*_r &= V_{rs}^r + \Phi^*_r \\
T^*_{em} &= \frac{1}{R_{s1} + sL_{s1}} V_{ds}^r \\
I^*_q &= \frac{1}{R_{s1} + sL_{s1}} V_{qs}^r
\end{align*}
\] (46)

![Fig. 6. Block diagram of the multivariable predictive control.](image)

\![Fig. 7. Block diagram of the MIMO predictive control of DSIM.](image)
Taking into account that the rotor flux is kept constant at its reference value, the substitution of (46) into (45) gives:

\[
\begin{align*}
\Phi^*_r & = \frac{2L_m}{(1 + T_s s)(R_{sl} + sL_{sl})} \Phi^r_v \\
T^*_{em} & = \frac{2pL_m}{(L_m + L_r)(R_{sl} + sL_{sl})} \Phi^r_v
\end{align*}
\]  
(47)

According to the system (12), the reference voltages are decomposed by two voltage vectors:

\[
\begin{align*}
V^*_{ds} & = V^*_{ds} - V^c_{ds} \\
V^*_{qs} & = V^*_{qs} - V^c_{qs}
\end{align*}
\]  
(48)

The rewriting of (47) in the matrix form gives:

\[
\begin{bmatrix}
\Phi^*_r \\
T^*_{em}
\end{bmatrix} = 
\begin{bmatrix}
G_{11}(s) & 0 \\
0 & G_{22}(s)
\end{bmatrix}
\begin{bmatrix}
V^r_{ds} \\
V^r_{qs}
\end{bmatrix}
\]  
(49)

The polynomials matrices of the CARIMA model (29) are:

\[
\begin{align*}
A(q^{-1}) &= \begin{bmatrix}
(1-0.84q^{-1} + 0.84q^{-2}) & 0 \\
0 & (1-0.84q^{-1} + 0.84q^{-2})
\end{bmatrix} \\
B(q^{-1}) &= \begin{bmatrix}
(8.95 + 8.44q^{-1})(1-0.84q^{-1}) \cdot 10^{-5} & 0 \\
0 & 0.082(1-0.84q^{-1} + 0.84q^{-2})
\end{bmatrix} \\
C(s) &= \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\end{align*}
\]  
(52)

The matrix form of the prediction equation (44) for \( j = 1 \) is:

\[
\begin{align*}
\Phi^r_{(k+1)} & = \begin{bmatrix}
1.84 -1.68q^{-1} + 0.84q^{-2} & 0 \\
0 & 1.84 -1.68q^{-1} + 0.84q^{-2}
\end{bmatrix}
\Phi^r_{(k)} + \\
& + \begin{bmatrix}
8.95 \cdot 10^{-5} - 7.0896 \cdot 10^{-5} q^{-2} & 0 \\
0 & 8.95 \cdot 10^{-5} - 7.0896 \cdot 10^{-5} q^{-2}
\end{bmatrix}
\begin{bmatrix}
\Delta V^r_{qs}(k) \\
\Delta V^r_{qs}(k)
\end{bmatrix} + \\
& + \begin{bmatrix}
8.4399 & 0 \\
0 & 8.4399
\end{bmatrix}
\begin{bmatrix}
\Delta V^r_{qs}(k-1) \\
\Delta V^r_{qs}(k-1)
\end{bmatrix}
\end{align*}
\]  
(53)

Where:
The multivariable cost function is defined by equation (42):
\[
J_{\text{GPC}} = U^T (H^T H + \lambda I) U + 2 U^T H^T (E - W) + (E - W)^T (E - W)
\]
(55)
The analytic minimization \( \frac{dJ_{\text{GPC}}}{dU} = 0 \) gives:
\[
\ddot{U} = (H^T H + \lambda I_N) \dddot{U} + H^T (W - E)
\]
(56)

With:
\[
W = \begin{bmatrix}
\Phi^*_e (k + 1) \\
\vdots \\
\Phi^*_e (k + j) \\
\vdots \\
\Phi^*_e (k + l)
\end{bmatrix},
U = U^* = \begin{bmatrix}
\Delta V^*_d (k) \\
\vdots \\
\Delta V^*_d (k + j - 1) \\
\vdots \\
\Delta V^*_d (k + l - 1)
\end{bmatrix}
\]
\[
E = G(q^{-1}) \begin{bmatrix}
\Phi_e (k) \\
\vdots \\
\Phi_e (k + j)
\end{bmatrix} + J(q^{-1}) \begin{bmatrix}
\Delta V^*_d (k - l) \\
\vdots \\
\Delta V^*_d (k + l - 1)
\end{bmatrix}
\]
and:
\[
G(q^{-1}) = \begin{bmatrix}
G_1(q^{-1}) & J_1(q^{-1}) \\
\vdots & \vdots \\
G_j(q^{-1}) & J_j(q^{-1})
\end{bmatrix}
\]
\[
H(q^{-1}) = \begin{bmatrix}
H_1(q^{-1}) & 0 & 0 & 0 \\
H_2(q^{-1}) & H_1(q^{-1}) & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots \\
H_j(q^{-1}) & \ldots & H_1(q^{-1}) & H_1(q^{-1})
\end{bmatrix}
\]

5.2. Synthesis of the SISO Predictive Control law
The prediction model is given by the following mechanical equation [13], [20]:
\[
\begin{bmatrix}
\Phi_e (k) \\
\Phi_e (k + j)
\end{bmatrix} = \begin{bmatrix}
\Phi_e (k) \\
\Phi_e (k + j)
\end{bmatrix} + \begin{bmatrix}
\Delta V^*_d (k) \\
\Delta V^*_d (k + j - 1)
\end{bmatrix}
\]
(54)

6. SIMULATION RESULTS
Using the block diagram of Fig. 7, the simulation was carried out under the same conditions as the conventional FOC control with the following adjustment parameters:
Table 1. Predictive control setting parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>MIMO_Predictive (Torque/Flux Norm)</th>
<th>SISO_Predictive (Speed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>N2</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Nu</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>λ</td>
<td>Matrix 0.01 I₅₂</td>
<td>0.002</td>
</tr>
</tbody>
</table>

6.1. Discussion

At start-up and during the transient regime (Fig. 8), the speed increases linearly as a function of time and reaches its reference value $288 \text{rd/sec}$ at $t = 0.75 \text{sec}$ without overshoot. The electromagnetic torque of the DSIM reaches its maximum value at start-up and then reaches the steady state at $t = 0.75 \text{sec}$. At the beginning, the stator current $I_{d1s}$ reaches a current of 33A. The stator voltage $V_{d1s}$ is sinusoidal in shape. The rotor flux norm presents at the start-up some peaks for a fraction of a second oscillating around their setpoints. At time $t = 2.5 \text{sec}$, the speed reverses and reaches its negative setpoint after $1.2 \text{sec}$ without any overshoot. This generates an increase in the current $I_{s1a}$ of a magnitude equal to that recorded during the start-up, which stabilizes after $1.2 \text{sec}$, the electromagnetic torque reaches its maximum value at the time of the inversion of the speed, which stabilizes once the latter reaches its setpoint ($-288 \text{rd/sec}$); the torque $T_{futur}$ progresses in a manner analogous to the reference torque; the flux norm $\Phi_{futur}$ follows their reference value during the inversion of the speed.

6.2. Robustness Test

Figs. (9.1) to (9.4) respectively show the no load characteristics of the DSIM with speed and flux norm regulation by a multivariable predictive control, followed by increasing the rotor resistance, stator resistance and decreasing the mutual inductance 75% of its nominal values, and an increase in the inertia 200% of its nominal value between $t = 1.5 \text{sec}$ and $t = 2 \text{sec}$.

The simulations results show clearly the insensitivity of the predictive control to the variation of the mutual inductance. On the contrary, we note that a 75% increase in the value of the rotor resistance, stator resistance and inertia, has a little influence on the performance. We notice (Fig. 10) a slight decrease in speed and a slight increase in the flux norm for a simultaneous variation of +75% Rₐ, +75% Rₛ, -75% Lₐ and +200% J.
The prediction operation of the speed, the flux and the torque increases reliability, reduces the complexity and the cost of the system. Indeed, the speed variation of the double star induction machine in variable speed range is a difficult problem to be overcome with respect to the parametric variation and in particular the mutual inductance, the rotor resistance and the stator resistors, thus causing the instability of the system.

From the obtained results, it can be concluded that the studied techniques are valid for the nominal conditions, even satisfying the operations in variable speed drive and even when the machine is loaded, on the other hand they have acceptable robustness to the variation of the load, thus achieving good static and dynamic performance.

8. APPENDIX

Double stator induction motor parameters [3], [4], [12], [20]

\[ P_r = 4.5kW, f = 50Hz, V_{in} = 220/380V, I_{in} = 6.5A, \]
\[ \Omega_r = 2751rpm, p = 1 \]
\[ R_s = R_r = 3.72\Omega, R_f = 2.12\Omega, L_s = 0.022H, L_f = 0.006H, L_m = 0.3672H \]
\[ J = 0.0625\text{ Kgm}^2, K_f = 0.001\text{ Nm(rad/s)}^{-1} \]

REFERENCES